Te Poutāhū
Curriculum Centre

Te Mātaiaho

The New Zealand Curriculum

MATHEMATICS AND STATISTICS YEARS 0–8

Mātai aho tāhūnui,
Mātai aho tāhūroa,
Hei takapau wānanga
E hora nei.

Lay the kaupapa down
And sustain it,
The learning here
Laid out before us.

Te Tāhuhu o te Mātauranga
Ministry of Education

Te Kāwanatanga o Aotearoa
New Zealand Government

OCTOBER 2024

Contents

There are two versions of the mathematics and statistics learning area. To ensure you can see the full teaching sequences, which spread across two pages:

- **for printing, print the 'single pages' version, backed; staple or bulldog clip the left-hand side of the printout**
- **for reading on line, view the 'double pages' version.**

The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The New Zealand curriculum is knowledge-rich. It prioritises and explicitly describes what must be taught each year and is deliberately sequenced to enable students to build knowledge, skills, and competencies systematically over time. It supports teachers to design teaching programmes that bring learning to life in the classroom, using local, national, and global contexts.

The science of learning informs curriculum sequencing and teaching practice. The curriculum builds on scientific understanding to identify five characteristics of how we learn:

We learn best when we experience a sense of belonging in the learning environment and feel valued and supported.

Students bring with them different cultural identities, knowledge, belief systems, and experiences. They need to see that these are valued and reflected in a school environment characterised by strong relationships and mutual respect. Students' sense of belonging is enhanced by sensitivity to their individual needs, emotions, cultures, and beliefs.

A new idea or concept is always interpreted through, and learned in association with, existing knowledge.

The amount of existing knowledge students have, and the degree to which that knowledge is interconnected in long-term memory, influence both the quality and ease with which they can build on that knowledge. Recognising and drawing on students' prior knowledge therefore improves their learning.

Establishing knowledge in a well-organised way in long-term memory reduces students' cognitive load when building on that knowledge. It also enables them to apply and transfer the knowledge.

Establishing new knowledge and skill in long-term memory requires active engagement and multiple opportunities to engage with them, practise them, and connect them to existing knowledge structures. When knowledge is well organised in long-term memory, students are more likely to be able to build on it and apply it in novel ways. If knowledge is not well established in long-term memory, students' working memory is likely to be overloaded when they attempt to build on or apply it. This cognitive overload can cause confusion, anxiety, and disengagement.

Our social and emotional wellbeing directly impacts on our ability to learn new knowledge.

Social and emotional wellbeing reduces anxiety, which frees cognitive capacity to learn new knowledge and skills, leading to deeper, more durable learning. Conversely, anxiety and negative emotions inhibit students' ability to learn. The factors that impact positively or negatively on social and emotional wellbeing vary between students. The influence of these factors is dynamic – it fluctuates over time, even during the course of a single day.

Motivation is critical for wellbeing and engagement in learning.

Motivation develops when students feel that three basic needs are met: autonomy – developing increasing self-direction in learning; competence – experiencing success in learning and seeing oneself as a successful learner; social connection – belonging and contributing to a group from which one learns. Success in learning helps to build motivation.

The New Zealand Curriculum – knowledge-rich, informed by the science of learning, and framed within the whakapapa of Te Mātaiaho

The design of this framework encompasses seven curriculum components. Te Mātaiaho as a whole weaves together these components, all of which begin with the word 'mātai', meaning to observe, examine, and deliberately consider.

Mātairangi | The guiding kaupapa

The overarching kaupapa guiding the curriculum, based on the science of learning and ensuring excellent and equitable outcomes for students

Mātai ki te rangi, homai te kauhau wānanga ki uta, ka whiti he ora. | Look beyond the horizon, and draw near the bodies of knowledge that will take us into the future.

The outer rings represent our guiding kaupapa.

Mātaiahikā | Relationships with tangata whenua and local community

Learning through relationships with tangata whenua and local communities

Mātai kōrero ahiahi. | Keep the hearth occupied, maintain the stories by firelight.

Poutama curves represent relationships with tangata whenua and the community.

Mātaioho | National curriculum – contextualised

The process by which schools bring the national curriculum to life through local, national, and global contexts

Mātai oho, mātai ara, whītiki, whakatika. | Awaken, arise, and prepare for action.

Unaunahi scales represent wealth of knowledge, purpose, and know-how.

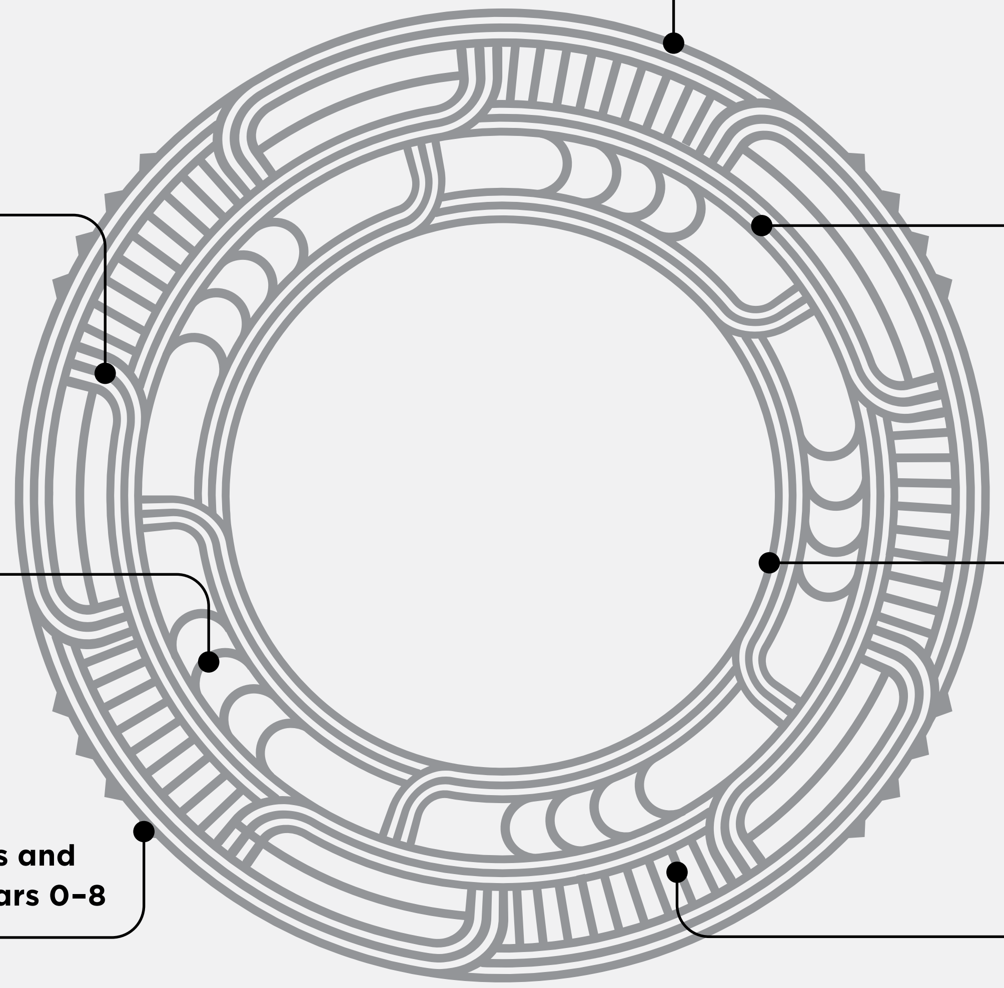
Mātaiaho | Learning areas

The eight learning areas, which each include a purpose, big ideas, knowledge, and practices, year-by-year

Mātai rangaranga te aho tū, te aho pae. | Weave the learning strands together.

Taratara-a-kae niho notches represent diversity, resilience, and mana.

› **Mathematics and statistics years 0–8**



Mātainuku | Creating a foundation

The curriculum principles (e.g., holding high expectations, and enabling all students to access the full scope of the curriculum)

Mātai ki te whenua, ka tiritiria, ka poupoua. | Ground and nurture the learning.

The centre rings represent the foundation and calls to action.

Mātaitipu | Vision of young people

The educational vision of young people, as conceived by young people

Mātaitipu hei papa whenuakura. | Grow and nourish a thriving community.

The inner rings and circular space represent the vision and students at the centre.

Mātairea | Supporting progress

The whole schooling pathway and the overarching focus for year-by-year learning and progress

Mātai ka rea, ka pihi hei māhuri. | Build and support progress.

Niho kurī lines represent building and supporting the development of students.

Learning areas

The curriculum has eight learning areas: English, the arts, health and physical education, learning languages, mathematics and statistics, science, social sciences, and technology. Together they provide the basis for a broad, general education for the first four phases of learning (years 0–10) and collectively lay a foundation for specialisation in phase 5 (years 11–13).

Each learning area is knowledge-rich. This knowledge has been carefully chosen to support all students in their schooling pathway and is framed using Understand, Know, and Do:

- **Understand** – the deep and enduring big ideas and themes that students develop understanding of over the phases
- **Know** – the meaningful and important content, concepts, and topics at each phase that enrich students' understanding of the big ideas and themes and that students study using the practices
- **Do** – the practices (skills, strategies, and processes) that bring rigour to learning and support the development of the key competencies.

A **progression model** provides the structure that sequences the knowledge. It supports all students to develop greater:

- breadth and depth of knowledge and understanding, through engaging with increasingly complex and ambiguous contexts
- refinement and sophistication in their use of competencies, practices, strategies, processes, and skills
- ability to connect, transfer, and apply new learning in meaningful contexts
- knowledge and awareness of themselves as learners
- effectiveness when working with others.

Content of the learning areas

Knowledge and progression are reflected in how the learning areas are organised. Each learning area has the following main sections:

Purpose statement and UKD overview

A purpose statement describes the learning area's contribution to the lives of students. It is followed by an overview of Understand, Know, and Do. This gives a view of the big ideas, themes, concepts, topics, and practices that underpin the learning area.

Teachers use the purpose statement and UKD overview to develop an understanding of the learning area, so that they can share its benefits with students.

Learning area structure

For each learning area, this section outlines its structure and the changes it undergoes over five phases of learning, particularly in the final phase, where students specialise and choose from a range of subjects.

There are five phases of learning, spanning years 0–13. Each phase covers two to three years of schooling, which reflects how most schools organise learning across year levels.

A **critical focus** for each phase establishes a sustained, strengths-based, focus on the student and their social, emotional, and cognitive learning at this stage of their schooling journey. Each critical focus builds on the phase before and is reflected in the content of the learning area for the phase.

The critical focuses are:

- **Phase 1** (years 0–3): Thriving in environments rich in literacy and maths
- **Phase 2** (years 4–6): Expanding horizons of knowledge, and collaboration
- **Phase 3** (years 7–8): Seeing ourselves in the wider world and advocating with and for others
- **Phase 4** (years 9–10): Having a purpose and being empathetic and resilient
- **Phase 5** (years 11–13): Navigating pathways and developing agency to help shape the future.

Teachers use the critical focus of each phase in their selection and design of topics and activities.

Teaching guidance

Each learning area also draws from the science of learning and wider education theory to provide a knowledge base and guidance for teachers. Teachers use this to help them make purposeful decisions about how to teach the learning area's content in ways that are inclusive of all students.

The guidance is organised under three headings:

- Designing a comprehensive teaching and learning programme
- Using assessment to inform teaching
- Planning.

Progress outcomes

In each learning area, there is one comprehensive progress outcome for each phase.

The progress outcomes act as signposts that describe expectations for what students should sufficiently understand, know, and be able to do at key points in the schooling pathway.

The content of each progress outcome is organised using the Understand–Know–Do framework. While the Understand statements repeat across the five phases, students' depth of understanding increases as their knowledge of the learning area's content (Know) grows and their use of the practices (Do) develops.

When read alongside the progress outcomes for prior and subsequent phases, the progress outcome for a phase helps teachers maintain an overview of the learning they are building on and the learning they are preparing students for. Progress outcomes are therefore key for planning, along with the more detailed teaching sequences (described below).

Teachers also use the progress outcomes to help them form a comprehensive view of each student's progress, achievement, learning needs, and strengths. Schools can use information from twice-yearly, standardised assessment tools to help develop this view, which can also be used to report to parents.

In forming a view of progress and achievement, teachers should ask themselves:

- **Are students using learning from the progress outcome of the previous phase to make sense of new learning in the current phase?** This demonstrates how well they can connect new learning to what they already know. It generally occurs in the first year of a phase.
- **Are students consolidating the learning expressed in the progress outcome in a wide range of contexts?** This demonstrates how well and confidently they are using their new learning. This generally occurs in the second year of the phase.
- **Are students secure in the learning described in the progress outcome within an increasingly complex range of contexts?** Are they showing greater depth of knowledge, understanding, and application as they use their new learning and prepare for the challenges of the next phase? This generally occurs towards the end of the final year of the phase.
- **Are there gaps in learning that are going to restrict students' ability to make progress in the next phase of their learning?** This is a question teachers should ask across all years of the phase, drawing on the section *Using assessment to inform teaching* (page 23) to consider how to adapt their practices to meet students' learning needs.

Leaders must have a mechanism and strategies for prioritising and closely monitoring urgent action, when required, to support classroom teaching. Where teaching needs to be targeted and intensified to meet specific needs for finite periods, leaders draw on a breadth of available supports, as required.

Teaching sequences

Each phase has a year-by-year teaching sequence. These sequences support teachers to know what to teach and when and how to teach it as students work towards the progress outcome for the phase. They have been organised to support students to revisit ideas, knowledge, and practices in ways that deepen their learning and enable them to use it at the next phase.

There are two parts in a teaching sequence: statements of **what** to teach, and 'teaching considerations' for **how** to teach:

- the 'what to teach' statements are preceded by the stem 'Informed by prior learning …', which reminds teachers to use their professional judgment and assessment information when selecting what content to teach
- the teaching considerations help teachers to know 'how to teach' this content in response to students' prior knowledge, strengths, and experiences.

The teaching sequence tables should be viewed both vertically and horizontally. Looking down the columns helps teachers know what to plan for in a year's programme. Looking across the rows at the statements for the same concept in the preceding and following years helps teachers to recognise prior learning that students may come with and to consider how they might extend this year's learning. It also helps teachers to form a more detailed view of their students' progress, and it is a strong support when planning for mixed-level classes.

The approach of the year-by-year teaching sequences changes in phase 5, as the content becomes more discipline-focused.

Te Mātaiaho

The New Zealand Curriculum

MATHEMATICS AND STATISTICS YEARS 0–8

Board requirements

Schools and kura must give effect to the learning area *Mathematics and statistics Years 0–8*.

Mathematics and Statistics Years 0–8 is published by the Minister of Education under section 90(1) of the Education and Training Act 2020 (the Act) as a foundation curriculum policy statement and a national curriculum statement. These are the statements of official policy in relation to the teaching of mathematics and statistics that give direction to each school's curriculum and assessment responsibilities (section 127 of the Act), teaching and learning programmes (section 164 of the Act), and monitoring and reporting of student performance (section 165 of the Act and associated Regulations). School boards must ensure that they and their principal and staff give effect to these statements.

The sections of *Mathematics and Statistics Years 0–8* that are published as a national curriculum statement are the Understand-Know-Do (UKD) progress outcomes for each phase (pages 27–29, 55–57, and 83–85). These set out what students are expected to learn over their time at school, including the desirable levels of knowledge, understanding, and skill to be achieved in mathematics and statistics.

The rest is published as a foundation curriculum policy statement. This sets out expectations for teaching, learning, and assessment that underpin the national curriculum statement and give direction for effective mathematics and statistics (or maths, including numeracy) teaching and learning programmes.

The statements come into effect on **1 January 2025**. They replace curriculum levels 1–4 of the existing mathematics and statistics national curriculum statement (learning area). The remainder of the existing mathematics and statistics national curriculum statement remains in force. Apart from those for *English Years 0–6*, other existing foundation curriculum policy statements and national curriculum statements for the New Zealand Curriculum remain in place.

Schools should choose the appropriate mathematics and statistics statements for their students' needs. This means that intermediate and secondary schools may choose to make use of the new statements for some students if they are currently working below curriculum level 5, or that primary and intermediate schools may choose to make use of the existing statements for some students if they are already working above phase 3.

Reading, writing, and maths teaching time requirements

The teaching and learning of reading, writing,[1] and maths[2] is a priority for all schools. So that all students are getting sufficient teaching and learning time for reading, writing, and maths, each school board with students in years 0–8 must, through its principal and staff, structure their teaching and learning programmes and/or timetables to provide:

- 10 hours per week of teaching and learning focused on supporting students' progress and achievement in reading and writing, and recognising the important contribution oral language development makes, particularly in the early phases of learning
- 5 hours per week of teaching and learning focused on supporting students' progress and achievement in maths.

Where reading, writing, and/or maths teaching and learning time is occurring within the context of national curriculum statements other than English or mathematics and statistics, the progression of students' reading, writing, and/or maths dispositions, knowledge, and skills at the appropriate level must be explicitly and intentionally planned for and attended to.

1 While the terms reading and writing are used, these expectations are inclusive of alternative methods of communication, including New Zealand Sign Language, augmentative and alternative communication (AAC), and Braille.

2 For simplicity, 'maths' is used as an all-encompassing term to refer to the grouping of subject matter, dispositions, skills, competencies, and understandings that encompasses all aspects of numeracy, mathematics, and statistics.

Purpose statement

Ānō me he whare pūngāwerewere.
Behold, it is like the web of a spider.

This whakataukī celebrates intricacy, complexity, interconnectedness, and strength. The learning area of mathematics and statistics weaves together the effort and creativity of many cultures that over time have used mathematical and statistical ideas to understand their world.

In the mathematics and statistics learning area, students learn about and appreciate the power of symbolic representation, reasoning, and abstraction. They learn to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. As they achieve deep conceptual understanding and procedural fluency in the learning area, students can accurately and efficiently use mathematics and statistics as a foundation for new learning and to solve problems.

Students engage with mathematics and statistics through the exploration of problems, patterns, and trends and appreciate the everyday value of this learning in many areas of their lives, such as personal finance, health, dance, and design. Every student in New Zealand can engage in mathematics and statistics and discover personal enjoyment and curiosity in their learning.

Throughout their learning, students engage with diverse perspectives as they apply their mathematical and statistical understandings. They also learn that mathematics and statistics has an evolving history; many cultures have contributed to, and continue to contribute to, innovations that shape our current thinking.

As they move through the phases of the learning area, students come to understand the value of mathematical and statistical investigation as a lens for collective local, national, and global challenges. Mathematics and statistics allow us to engage with important societal matters, such as the robust and ethical gathering, interpretation, and communication of data, and the use of valid and reliable data to challenge misinformation and disinformation.

Learning in mathematics and statistics builds literacy by developing students' skills in oral and written communication, reasoning, and comprehension. The learning area opens pathways into a wide range of industries that rely on mathematical and statistical knowledge and reasoning. Learning how to use this knowledge purposefully and flexibly allows students to participate fully in an increasingly technology- and information-rich world of work.

Understand-Know-Do Overview

Understand

Understand describes the deep and enduring mathematical and statistical **big ideas** that students develop over phases 1–5.

Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

Know

Know describes the meaningful and important mathematical and statistical **concepts and procedures** through which students develop understanding of the big ideas.

Number | Mātauranga tau

Number focuses on the study of numerical concepts. People use numbers to represent quantities, estimate, and measure. We perform operations on numbers to calculate or compare. Throughout history, different number systems have been developed, reflecting practical and social needs.

Algebra | Taurangi

Algebra focuses on making and using generalisations to reason mathematically. It allows us to identify patterns and underlying mathematical relationships. These generalisations, patterns, and relationships can be represented and communicated using diagrams, graphs, and symbols (including variables). The algebra we use today was created and refined over thousands of years.

Measurement | Ine

Measurement focuses on the concepts and techniques that allow us to quantify phenomena, using appropriate units and systems of measurement. Countries around the world use both standard and non-standard units to measure tangible and intangible objects and quantities.

Geometry | Āhuahanga

Geometry focuses on visualising, representing, and reasoning about the shape, position, orientation, and transformation of objects. Many cultures use tools and techniques derived from the natural world when exploring and describing objects and space.

Statistics | Tauanga

Statistics focuses on tools, concepts, and systematic processes for interpreting situations, using data and its context to understand uncertainty, make conjectures, and inform decision making. Statistical practices include considering the ethics of data collection and the responsibility of safely and securely handling data in different contexts.

Probability | Tūponotanga

Probability focuses on tools and concepts for quantifying chance, dealing with expectation, and using evidence to identify how likely events are to occur. People around the world have relied on and continue to rely on probabilistic thinking when making decisions.

Do

Do describes the **processes** that are fundamental to all mathematical and statistical activities and that underpin students' learning of the big ideas, concepts, and procedures.

Investigating situations | Te tūhura pūāhua

When we investigate situations using mathematics and statistics, we describe and explore them to build our understanding of them. When investigating, we need to decide which approaches, concepts, and tools to use and how to use them. We often begin with a question or focus of interest and proceed in systematic but flexible ways, using mathematical and statistical concepts and procedures to solve problems and make sense of findings in context. We conclude by evaluating the investigation, which involves reflecting on the solutions and outcomes and our approaches and choices to determine whether they were reasonable, made sense in context, and could be improved on in future investigations.

Representing situations | Te whakaata pūāhua

When we represent situations mathematically and statistically, we use words or symbols and mental, oral, physical, digital, graphical, or diagrammatic ways to show concepts and findings. We can use representations to compare, explore, simplify, illustrate, prove, and justify, as well as to look for patterns, variations, and trends. Representing a situation in multiple ways enables a deeper and more flexible understanding and allows us to communicate with different audiences.

Connecting situations | Te tūhono pūāhua

When we connect situations using mathematics and statistics, we recognise and make links by noticing similarities and differences. Connecting helps us to understand the relationships between concepts and procedures in mathematics and statistics. This is important because number, algebra, measurement, geometry, statistics, and probability form a web of interconnected ideas and approaches that can be easier to remember and understand if the connections between them are clear. Connecting also involves linking mathematics and statistics to other learning areas and to a range of contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

When we generalise mathematical and statistical findings, we move from specific examples to general principles. We use the patterns, regularities, and structures that we find to make conjectures that might apply to other situations. Further investigation can test and refine these conjectures and determine if they apply in all cases. In statistics, we generalise by using trends and variation in data to make inferences and conjectures and to articulate and evaluate claims about similar situations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

When we explain and justify, we use mathematical and statistical ways of communicating and reasoning to share our ideas and to respond to the ideas, reasoning, and inferences of others. Explaining is how we communicate our inferences and conjectures, build arguments, and unpack stories from data. Justifying involves describing why decisions and findings are reasonable, taking into account limitations arising from assumptions and choices and the evidence on which findings are based.

Mathematics and statistics learning area structure

This section describes the structure of the mathematics and statistics learning area and how it changes over the five phases of learning. (See pages 9–11 for the general structure of each learning area in the New Zealand curriculum.)

Each phase has:

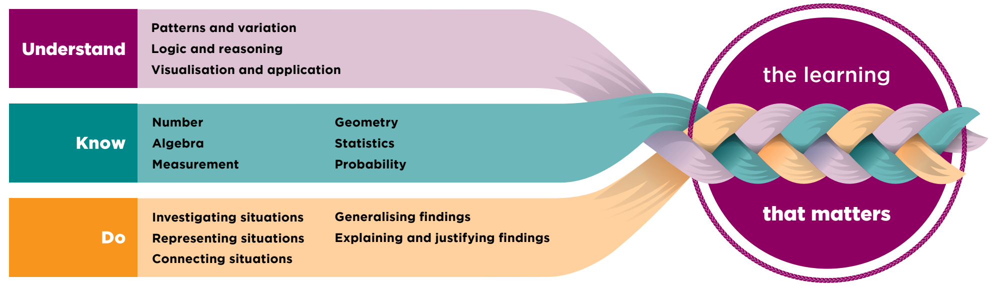
- a progress outcome describing what students understand, know, and can do by the end of the phase
- an introduction to the teaching sequence highlighting how to teach during this particular phase
- a year-by-year teaching sequence highlighting what to teach in the phase, along with teaching considerations for particular aspects of content.

Progress outcomes

The progress outcomes (one per phase) describe what students will understand, know, and be able to do by the end of the phase.

- **Understand** describes the big ideas that students develop from learning mathematics and statistics over phases 1 to 5. They help connect school mathematics and statistics with the wider world and represent the critical big-picture concepts of mathematics and statistics.
- **Know** describes the meaningful and important concepts and procedures in mathematics and statistics. They are broken down into six strands: number, algebra, measurement, geometry, statistics, and probability.
- **Do** describes the processes students use to represent and work with what they know and understand in mathematics and statistics. These processes are central to how students learn and apply mathematical and statistical knowledge. While there are small progressions in the processes from phase to phase, in general the increasing sophistication of their use comes from applying them to more advanced concepts and procedures.

It is through the interweaving of Understand, Know, and Do that students develop their conceptual understandings and procedural fluency, supporting success and bringing richness and meaning to mathematics and statistics for them.



As students progress through the phases, the focus of their learning shifts. In phase 1, the focus is on developing foundational skills across all strands. In phases 2 and 3, students expand their range of representations and their reasoning to work with increasingly complex concepts across all strands.

This change in focus is seen in how the Understand, Know, and Do progress outcomes are reflected in the year-by-year teaching sequences. The descriptors of what to teach each year have the stem 'Informed by prior learning …' in order to reinforce that teachers will use their professional judgment about what content to teach and how to teach it. They will make these judgments in response to the prior knowledge, strengths, and experiences that students bring to their learning.

Teaching sequences

The year-by-year teaching sequences are organised in line with the strands from Know. They describe the incremental teaching required each year as students work towards the progress outcome.

Some statements in the teaching sequences are repeated across multiple years, allowing more time for progression and consolidation. Not all statements are progressed each year; some topics start and others end, reflecting what is developmentally appropriate in learning in mathematics and statistics.

Each statement in a sequence varies in the amount of teaching time it requires. The learning area is designed to enable knowledge and procedures to be connected and taught together, so individual statements in a year sequence should be combined in ways that enhance learning.

The year-by-year content can be viewed both vertically and horizontally. The vertical view helps teachers know what to plan for the next year. The horizontal view allows teachers to follow the statements for one concept across several stages. This helps them understand the prior knowledge students may bring to their learning and helps them decide how to extend this learning. The horizontal view also helps teachers plan for mixed-level classes.

The teaching sequence statements are supported by 'teaching considerations'. These describe evidence-based practices and show how teachers can integrate the processes of Do to help their students develop conceptual and procedural knowledge.

Teaching guidance

Key characteristics of how people learn have informed the development of the mathematics and statistics learning area. These characteristics are:

- We learn best when we experience a sense of belonging in the learning environment and feel valued and supported.
- A new idea or concept is always interpreted through, and learned in association with, existing knowledge.
- Establishing knowledge in a well-organised way in long-term memory reduces students' cognitive load when building on that knowledge. It also enables them to apply and transfer the knowledge.
- Our social and emotional wellbeing directly impacts on our ability to learn new knowledge.
- Motivation is critical for wellbeing and engagement in learning.[1]

All five characteristics are interconnected in a dynamic way. They are always only pieces of the whole, so it is critical to consider them all together. The dynamic and individual nature of learning explains why we see individual learners develop along different paths and at different rates.

The implications of these characteristics for teaching mathematics and statistics are described in this section, with more detail in the introduction to each phase and the 'teaching considerations' in the year-by-year teaching sequences.

The remainder of this section focuses on three key areas of teacher decision making:

- developing a comprehensive teaching and learning programme
- using assessment to inform teaching
- planning.

1 A description of each characteristic is found on page 5.

Developing a comprehensive teaching and learning programme

A comprehensive mathematics and statistics programme needs the following components:

- explicit teaching
- positive relationships with mathematics and statistics
- rich tasks
- communication in mathematics and statistics.

Explicit teaching

Explicit teaching is a structured, carefully sequenced approach to teaching. The sequencing of content is thought out and broken down into manageable steps, each of which is clearly and concisely explained and modelled by the teacher. Explicit teaching requires a high level of teacher-student interaction, guided student practice, and, when proficiency is achieved, independent practice.

Explicit teaching supports cumulative learning as new knowledge is built on what students already know. Teachers provide multiple opportunities for practising, reviewing, consolidating, and using previous learning alongside new learning.

Explicit teaching takes account of cognitive overload. With sufficient practice, new learning is transferred to long-term memory. This frees up working memory, opening up opportunities for extension, enrichment, and new learning.

Explicit teaching is strongly interactive – it is not simply teacher talk. It includes rich discussions between teachers and students and amongst students, to check on understanding. Teachers adapt the pace of their teaching in response to students' progress. They engage students in creative and challenging tasks to foster motivation and engagement.

Using materials and visual representations throughout explicit teaching supports students to develop conceptual understandings as they move towards more abstract forms of representation, such as equations. Teachers can reduce students' cognitive load by carefully considering the ways in which visual and written information are presented (e.g., how working and explanations are laid out) and by removing unnecessary information to focus on the key teaching and learning points.

Explicit teaching involves:

- connecting the current focus to previous learning
- providing concise, step-by-step explanations, accompanied by student input and discussion
- explaining, modelling, and demonstrating
- regularly checking for understanding and providing feedback
- providing opportunities for collaborative and independent practice.

Positive relationships with mathematics and statistics

Learning is enhanced when students succeed in and feel positive about their learning. If students feel anxious, they have fewer cognitive resources available for learning.

Positive relationships with mathematics and statistics are supported by teachers through:

- setting high expectations
- planning experiences that are accessible to every student and provide daily opportunities for success
- incorporating students' interests, cultures, and prior knowledge
- planning opportunities for students to explore and think critically
- supporting students to use mathematics and statistics to make sense of their world and address local, national, and global issues
- providing manageable challenges that encourage students to develop perseverance, reinforcing that conceptual understanding and procedural fluency develop with consistent effort
- increasing scaffolding and supports in response to anxiety as a result of cognitive overload
- valuing mistakes as an important part of the learning process.

Involving families in students' learning journeys and offering opportunities for collaboration support positive relationships with mathematics and statistics. Teachers also model such relationships by showing curiosity, persistence, and enjoyment, and by engaging in mathematics and statistics themselves.

Rich tasks

Rich tasks are meaningful problem-solving and investigation experiences, designed to invoke curiosity and engagement. They should relate both to mathematical contexts and wider contexts relevant to the communities, cultures, interests, and aspirations of students.

Rich tasks provide a motivational hook when exploring new concepts and procedures. They can also be used to consolidate concepts and procedures that have already been taught, to develop the mathematical and statistical processeses of Do, and to facilitate the transfer and application of learning to new situations. These experiences often allow students to decide how to approach the task, developing their agency, confidence, and motivation.

Teachers design rich tasks that are accessible to all students and offer different levels of challenge. They ensure that students are clear about the purpose of learning, and they consider the core requirements of the task as well as the range of possible responses. As students work on rich tasks, teachers plan opportunities for discussion, collaboration, and feedback. They are actively involved in monitoring, prompting, and questioning during the task, to encourage students to ask questions, test conjectures, make generalisations, and form connections.

Communication in mathematics and statistics

Students communicate throughout the learning process, both to develop conceptual understanding and to share their thinking and reasoning. Rich, extended interactions are pivotal to students' development of knowledge, processes, and dispositions in mathematics and statistics. Effective discussions build knowledge through sharing, active listening or attending, critiquing, questioning, and extending thinking and reasoning.

Rich interactions make students' reasoning visible. This helps teachers recognise how well students are developing mathematical and statistical processes and concepts, and it provides opportunities for teachers to identify misconceptions and correct them. These interactions also allow teachers to develop students' use of mathematical and statistical language, vocabulary, symbols, representations, and reasoning.

Using assessment to inform teaching

Assessment that informs decisions about adapting teaching practice is moment-by-moment and ongoing. Teachers use observation, conversations, and low-stakes testing to continuously monitor students' progress in relation to their year level in the teaching sequence. They ensure that they notice and recognise the development, consolidation, and use of learning-area knowledge by students within daily lessons, and that they provide timely feedback. They respond by adapting their practice accordingly. For example, they reduce or increase scaffolding and supports, paying particular attention to anxiety caused by cognitive overload. Formative assessment information can also be collected through self and peer assessment, with students reflecting on goals and identifying next steps.

In addition to daily monitoring, teachers use purposefully designed, formative assessment tasks at different points throughout a unit or topic to highlight the concepts and reasoning students use and understand. Teachers ensure such tasks are valid by addressing barriers to learning, so that every student is able to demonstrate what they know and can do.

When planning next steps for teaching and learning, teachers consider students' strengths and responses along with potential opportunities for further consolidation. Next steps could include:

- designing scaffolds to support students to access and enrich their learning
- providing opportunities for students to apply new learning
- planning lessons focused on revisiting, reteaching, or consolidating learning.

Providing timely feedback throughout the learning process and identifying and addressing misconceptions as they arise lead to the efficient and accurate development of learning-area concepts and promote further learning. Teachers can use feedback to prompt students to recall previous learning, make connections, and extend their understanding.

Planning

This section provides guidance on what to pay attention to when planning mathematics and statistics teaching and learning programmes. In every classroom, there are many ways in which students engage in learning and show what they know and can do. Using assessment information and designing inclusive experiences, teachers plan an 'entry point' to a new concept or procedure that every student can access. Students' interests and the school culture and community shape the planning, adding richness, creativity, and meaning to the programme.

Teaching and learning plans are developed for each year, topic or unit, week, and lesson and make optimal use of instructional time. The following considerations are critical when planning and designing learning:

- Develop plans using the teaching sequence statements for the year and knowledge of students' prior learning. Plan for all students to experience all the statements in the sequence for their year level.
- Map out a year-long programme composed of 'units' by looking for opportunities where statements from the teaching sequence can be taught together. These may be in the same strand or across several strands (e.g., statistics and measurement; algebra and geometry). Plan to weave together learning under Know and Do across the unit to build understanding of the big ideas.
- Order the units so that new learning builds on students' previous learning and connects over the course of the year. Consider the length of time allocated to specific strands and concepts across the year – some concepts may require more teaching time than others. Ensure the year's programme includes opportunities to retrieve, consolidate, and extend learning around previously taught concepts and processes. Regular opportunities to revisit learning within and across units and years supports students to develop procedural fluency with mathematics and statistics concepts. The shape of these opportunities will vary, depending on students' learning needs.
- Within unit or weekly plans, break down new concepts and procedures into a series of manageable learning experiences, so that students have several opportunities to develop understanding and fluency. Teach mathematics and statistics for an hour a day. Plan for a balance of explicit teaching (to introduce and reinforce learning) and rich tasks (to investigate a concept, support consolidation of previously taught concepts or procedures, and apply learning to new situations).

- Plan for inclusive teaching and learning at all times. Consider offering multiple methods of participating to all students so that they can engage in a variety of learning experiences and have multiple ways to show their progress. Design for equitable access in all learning opportunities. Identify and reduce barriers to learning, and plan universal supports that are available to all students.
- Use flexible groups within a lesson, based on the learning purpose for the lesson (e.g., working as a whole class for demonstration and discussion, in smaller groups to investigate a situation or solve a problem, in pairs to explain thinking and findings). Provide opportunities for both individual and collaborative work, and enable students to determine when they need to work with others and when they need time and space to work independently.
- Teach students to use digital tools accurately, appropriately, and efficiently to support their purpose. Enhance teaching and learning with tools for calculating, representing graphs and shapes, and analysing data. While using digital technology is an important skill, students still need the ability to estimate, visualise, and reason, so that they can evaluate whether findings generated by a digital tool are reasonable and effective.

To support students who have not developed the prior knowledge needed for teaching sequence statements for their year or have not learnt everything they have been explicitly taught, teachers can use accelerative approaches. These are approaches that make year-level concepts and procedures accessible to students. They can include additional, targeted small-group teaching, the use of verbal and visual prompts, carefully chosen representations, and explicit teaching of problem-solving strategies.

Teachers can extend students who have developed deep conceptual understanding and procedural fluency for their year by using more challenging rich tasks and problem solving that allow the students to apply their understanding to unfamiliar situations. This also encourages the students to develop further generalisations and to strengthen their mathematical and statistical communication and reasoning.

Dedicated mathematics and statistics lessons

Depending on the purpose of the lesson, plan to include one or more experiences in each part (Getting started, Working, and Connecting and reflecting). As students are working, take time to notice, recognise, and respond to their learning.

Getting started	› Recall and connect to prior learning to provide a starting point for all students to access and understand new concepts or processes. › Introduce new concepts using a focus activity, group challenge, or task that activates prior knowledge and interests.
Working	› Provide whole-class, small-group, paired, or individual work opportunities for students to develop or apply concepts and procedures through investigations, tasks, or games. › Explicitly teach concepts and procedures by leading interactions that include explanations, demonstrations, questioning, short tasks, and discussion. Use clear and concise language, including correct mathematical and statistical vocabulary, and clear working layouts and notation. › Provide additional explicit teaching based on the learning needs of individual students. › Help students organise new knowledge in ways that connect with their prior learning – for example, by discussing connections, using graphic organisers, or carefully ordering concepts and procedures in relation to prior learning. › Support consolidation of knowledge with targeted practice and activities. For students early in the process of consolidation, these activities should be scaffolded and guided. As students develop understanding and fluency, they complete the activities with increasing independence. › Support students to retrieve and use previously taught concepts and procedures in connected ways, such as applying them while investigating situations.
Connecting and reflecting	› Clearly summarise and connect to the purpose of the lesson. › Review learning by discussing, sharing, and analysing the experiences of the lesson. › Make connections with prior learning, between mathematics and statistics concepts, with other learning areas, and with situations outside of the classroom. › Pre-teach to prepare students for the next lesson. › Highlight progress and examples of curiosity, resilience, and persevering through challenge.

Phase 1 Years 0–3

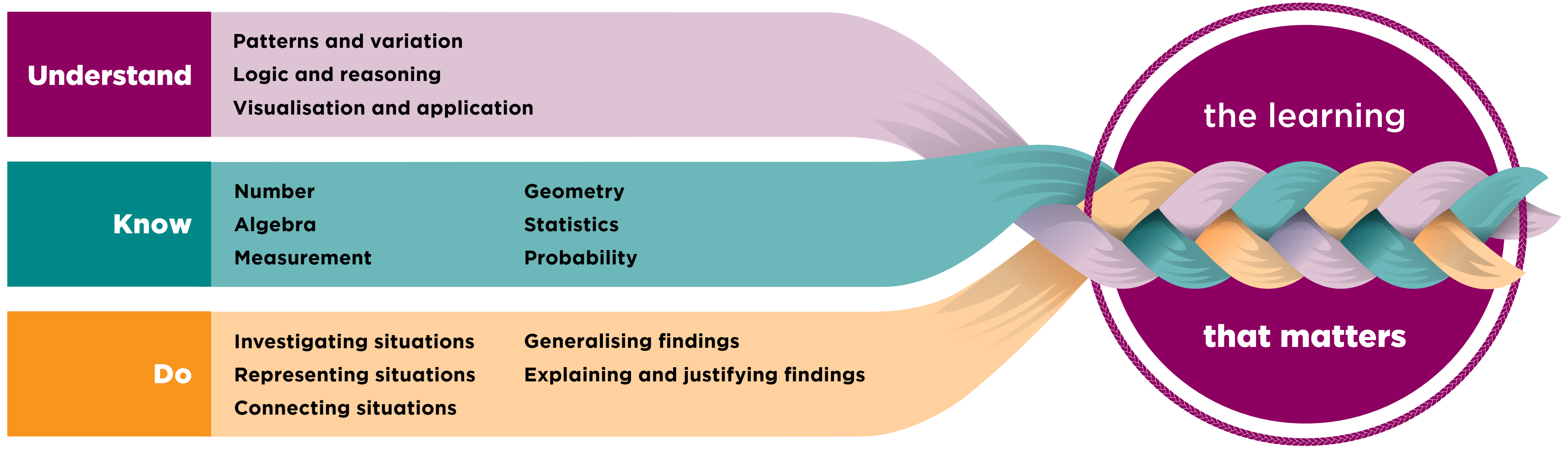
Progress outcome by the end of year 3 (Foundation)

Thriving in environments rich in literacy and maths

Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau

The critical focus of phase 1 is for all students to thrive in environments rich in literacy and maths. In mathematics and statistics, students learn to use logic and reasoning to investigate, classify, and describe patterns and variations in quantities, shapes, and data. They begin to generalise and to understand the properties of numbers and attributes of shapes. They use materials, number lines, and pictures to visualise these concepts, make connections between representations, and explain their reasoning.

The phase 1 progress outcome describes the understanding, knowledge, and processes that students have multiple opportunities to develop over the phase.



The phase 1 progress outcome is found on the following two pages.

Understand

As students build knowledge through their use of the mathematical and statistical processes, they begin to understand the following.

Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

Know

Number | Mātauranga tau

By the end of this phase, students know that our number system is base 10, with ten digit symbols. The place value of a digit in a number depends on its position; as we move to the left, each column is worth ten times more, with zero used as a placeholder. Students know that they can subitise (recognise without counting) patterns to support estimations and calculations. They know that numbers can be partitioned and recombined in different ways. Addition is putting parts together to find a total or whole. Subtraction takes parts away from a whole; it is also the difference between numbers. Multiplication and division involve recognising and working with equal groups and how many are in each group, the number of groups, and the total amount.

Students come to know that fractions are numbers that can be represented using words, pictures, or symbols. When fractions are represented symbolically, the bottom number (the denominator) shows how many pieces a whole has been equally split into, and the top number (the numerator) shows how many of those parts the fraction represents. Fractions show parts of a whole region, set of objects, or measurement.

Algebra | Taurangi

By the end of this phase, students know that patterns are made up of elements, including numeric or spatial elements, in a sequence governed by a rule. Repeating patterns have a unit of repeat; growing patterns can increase or decrease. The equal sign is relational in that it shows that the two sides of an equation represent the same quantity. Students also know that an algorithm is a set of step-by-step instructions for completing a task or solving a problem.

Measurement | Ine

By the end of this phase, students know that systems of measurement have a history and that different cultures use different approaches to measuring. Students know that they can measure and compare various attributes, such as length, area, volume, capacity, mass (weight), temperature, time, duration, and turn, using informal or standard units. When measuring, the measurement units must remain the same and there must be no gaps or overlaps between them. Students also know that the distance around the boundary of a two-dimensional shape gives perimeter, covering a surface gives area, and filling a three-dimensional shape gives capacity or volume.

Geometry | Āhuahanga

By the end of this phase, students know that patterns in shapes can be used to compare, classify, and predict. Two- and three-dimensional shapes have features that can be observed and described using geometric language. Shapes and objects can flip (reflect), turn (rotate), slide (translate), and be used to create patterns. Objects can be rotated in space and may appear different from other perspectives. Students know that maps are two-dimensional representations of places in the real world with symbols to show locations and landmarks. The position of a location can be described relative to another location, including a known environmental feature.

Statistics | Tauanga

By the end of this phase, students know that data is information about the world, that it comes in many forms, and that it helps them to learn about people, their lives, and their environment. They know that a statistical enquiry cycle can be used to investigate a group, using questions that they ask of the data for the group. A variable refers to an attribute or measurement of the people or objects being studied, such as colour, height, or number of children. Sorting and organising the data for variables helps to make sense of data and to answer summary investigative questions. Data visualisations are representations of all available values for one or more variables that reveal relationships or tell a story.

Probability | Tūponotanga

By the end of this phase, students know that a chance-based situation has a set of possible outcomes that can be arranged into events. The probability of an event is the chance of it occurring.

Do

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can work with others to pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it. They can identify relationships and relevant prior experience and knowledge to support the investigation. They can describe progress on the investigation pathway and work with others to make sense of outcomes or conclusions in the light of a given situation and context.

Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to explore, find, and illustrate patterns. They use representations to learn new ideas and explain ideas to others, and they use visualisation to mentally represent and manipulate groups and shapes. They select or create appropriate mental, oral, physical, or virtual representations.

Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They also make connections with ideas in other learning areas and with familiar local contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use words and pictures to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations about what they notice and wonder, and they make deductions based on prior knowledge. They ask questions to clarify and understand others' thinking and use evidence and reasoning to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others and can present basic explanations and arguments for an idea, solution, or process.

Phase 1 Years 0–3

Teaching sequence

Thriving in environments rich in literacy and maths
Te tupu pāhautea i te taiao ako e haumako ana i te reo matatini me te pāngarau

This section describes how the components of a comprehensive mathematics and statistics teaching and learning programme are used during the first phase of learning at school.

Throughout phase 1, students experience teaching that encourages curiosity and fosters success, as they explore environments and contexts rich in number and spatial elements. Active, hands-on experiences engage them in mathematics and statistics, with meaningful tasks that reflect their interests and the world outside the classroom.

Continuously monitor students' reasoning, questions, engagement, and use of representations, and respond quickly to address any misconceptions. Be mindful of providing manageable learning experiences, building on students' prior learning and leading to further challenge.

Explicit teaching

- Engage students in the mathematical and statistical processes of Do. Explicitly teach students to use them, and demonstrate them regularly as part of the teaching.
- Teach connected concepts and procedures together. For example, when teaching time (within the measurement strand) connect with fractions (within number) and turns (within geometry). Point out connections within concepts (e.g., "If I know 3 + 4, then I know 4 + 3").
- Demonstrate new concepts or procedures using clearly explained, manageable steps.
- Think 'aloud'. Voice decision making (e.g., about which numbers or operations to use) while demonstrating a procedure or process.
- Ensure that every student engages in the active recall of previous learning (e.g., through games, matching activities, 'think, pair, share'). Prompt students to make connections between previous and new learning.
- Plan ways for students to consolidate their mathematical and statistical learning and build procedural fluency. Use a range of guided and independent practice tasks, such as working on problems that use a procedure that has been demonstrated. Use songs, games, materials, families of facts, and digital tools to build fluency and for students to practise skip counting, addition, subtraction, multiplication, and division facts.

Positive relationships with mathematics and statistics

- Encourage students to 'have a go' and take risks. Reinforce the idea that mistakes help us learn as we try new procedures or share ideas.
- Select highly interesting contexts based on knowledge of students' personal experiences and backgrounds. Encourage students to connect with mathematics and statistics outside school by bringing in photos, resources, books, and other artefacts from home that link to mathematics and statistics learning.

Rich tasks

- Use open-ended investigations with the whole class, groups, or individuals to support students to understand concepts and extend their learning. For example, plan investigations into local situations (e.g., "What should the new items on the lunch order menu cost?") and into mathematical situations (e.g., the different ways of partitioning 24 into smaller groups).
- Choose problems or investigations that help students notice structures and relationships (e.g., present and discuss 'odd-one-out' numbers or shapes).
- Teach problem-solving and investigation strategies. Support students to read and make sense of a problem – through drawing, using materials, or trying some numbers – and to then plan how to solve it, take action to apply their plan, and check their findings.

Communication in mathematics and statistics

- Use numbers, materials, and pictorial representations (e.g., diagrams and pictures). Select representations that support the purpose of learning and help students to show their thinking and reasoning and to learn new ideas. Over the phase, move students towards using symbols and showing operations as equations. Number lines are a key representation in this phase for showing, ordering, and comparing numbers (including fractions) and for demonstrating operations.
- Prompt students to visualise and identify patterns, connections, and structures. Engage them in tasks where they are sorting, grouping, partitioning, and discussing what they have noticed and are wondering about. Guide them to notice and respond to patterns, similarities, and differences.
- Build students' mathematical and statistical vocabulary. Use games, songs, word walls, books, and digital tools. Intentionally use vocabulary to connect students' informal language with appropriate mathematical and statistical language. In doing so, draw on students' first and heritage languages, so that they can use their languages as a resource to connect their thinking and learning.
- Foster interactions that allow students to discuss, clarify, and explain their mathematical and statistical ideas. Encourage students to summarise, ask questions, and make suggestions. Help them to recall and connect mathematical and statistical learning using questions, materials, and verbal or visual prompts.

Number

	During the first 6 months *Informed by prior learning, teach students to:*	During the first year *Informed by prior learning, teach students to:*	During the second year *Informed by prior learning, teach students to:*	During the third year *Informed by prior learning, teach students to:*	Teaching considerations
Number structure	› subitise (recognise without counting) the number of objects in a collection of up to 5	› subitise (recognise without counting) the number of objects in a collection of up to 10, including by combining two patterns of 1–5 objects	› group objects in a collection of at least 10, subitise the number of objects in each part, and find the total number in the collection using the parts	› estimate the number of objects in a collection of less than 100, using patterns and groupings	Use a range of materials and images that **represent** structured and unstructured patterns and collections (e.g., dot patterns, 10s frames, dice, materials that can be grouped in 10 such as ice-block sticks). Also use language that quantifies and compares pattern arrangements (e.g. more, less, the same, different, combine, separate). **Connect** subitising to partitioning collections of objects (e.g., 6 and 2 on two dice are the same as 5 and 3 on two 10s frames).
	› count forwards or backwards from any whole number between 1 and 10, and then between 1 and 20	› count forwards or backwards in 1s, 2s, and 10s from any whole number between 1 and 20, and then between 1 and 100	› count forwards or backwards in 1s, 2s, 5s, and 10s from any whole number between 1 and 100	› count forwards or backwards in 2s, 3s, 5s, and 10s from any whole number between 1 and 1,000	Use a range of materials (e.g., number lines, 100s boards, number flip charts, 1,000s books, a Slavonic abacus, ice-block stick bundles). In general, support students to practise counting (e.g., in 2s and 5s) in short sequences (e.g., at year 3, "Count in 1s from 895 to 904; count in 2s from 90 to 110"). **Investigate** short patterns in multiples of 2s, 3s, 5s, and 10s, using rhymes, songs, choral counting, the grouping of discrete objects, the recording of patterns, and picture books. Have students practise finding 1, 10, or 100 more or less for a given number. Use materials to support students to identify numbers and patterns (e.g., 100s boards, 1,000s books). **Connect** to te reo Māori to support place-value (PV) understanding (e.g., tekau mā tahi (10 and 1), toru tekau mā rua (30 and 2).
	› identify, read, and write whole numbers up to at least 10	› identify, read, and write whole numbers up to at least 20, and represent them using the ten-and-ones structure of teen (11-19) and -ty (multiples of 10) numbers (e.g., 17 = 10 + 7, 20 = 2 × 10)	› identify, read, and write whole numbers up to at least 100, and represent them using base 10 structure	› identify, read, and write whole numbers up to at least 1,000, and represent them using base 10 structure	Have students practise saying, reading, and writing any given number within an identified number range. Use materials to support this (e.g., number flip boards, PV flip charts and houses). **Explain** that base 10 structure is based on groups of ten (ten ones form one ten, ten tens form one hundred, ten hundreds form one thousand etc.) and that both the position and value of a digit indicate the quantity it represents (e.g., 64 has 6 tens and 4 ones, 60 + 4 = 64). Have students **investigate** and **represent** the base 10 structure of numbers using a range of materials and digital tools (e.g., 100s boards, PV houses, PV blocks, ice-block sticks, arrow cards, number fans, words, numerals). **Investigate** odd and even numbers and the patterns they notice. **Connect** numerals, representations of them, and language (e.g., 652 represented with PV money: "652 = 600 + 50 + 2, 6 hundreds + 5 tens + 2 ones, six hundred and fifty two").
	› compare and order whole numbers up to at least 10 and ordinal numbers (e.g., 1st, 2nd, 3rd), using words	› compare and order whole numbers up to at least 20 and ordinal numbers (e.g., 1st, 2nd, 3rd), using words or numerals and suffixes	› compare and order whole numbers up to at least 100	› compare and order whole numbers up to at least 1,000	Show the sequencing of numbers using a number line (select-numbered, marked, or empty). Change the number-line orientation from horizontal to vertical if students need support with the concepts of before and after. **Explain** and use the language of comparison when demonstrating why one number is larger or smaller than another (e.g., "63 is larger than 36, as 6 tens is larger than 3 tens"). Show how the position of digits in the PV structure helps us to order and compare two- and three-digit numbers.

	During the first 6 months *Informed by prior learning, teach students to:*	During the first year *Informed by prior learning, teach students to:*	During the second year *Informed by prior learning, teach students to:*	During the third year *Informed by prior learning, teach students to:*
Number structure	› partition up to 5 objects, and then up to 10 objects, using a systematic approach and noticing patterns	› partition and regroup up to 20 objects in different ways, using a systematic approach and noticing patterns	› partition and regroup whole numbers up to at least 100, using a systematic approach and noticing patterns (e.g., 10 + __ = 70, 20 + __ = 70, 30 + __ = 70)	› partition and regroup whole numbers up to at least 1,000, using a systematic approach and noticing patterns (e.g., 400 + 300 = __, 350 + __ = 500)
Operations		› use estimation to predict results and to check the reasonableness of calculations	› use estimation to predict results and to check the reasonableness of calculations	› use estimation to predict results and to check the reasonableness of calculations
			› identify the nearest ten to any whole number up to 100	› round whole numbers up to 1,000 to the nearest hundred or ten
	› join and separate groups of up to a total of 10 objects by grouping and counting	› join and separate groups of up to a total of 20 objects and find the difference between groups by grouping and counting (e.g., 9 + 6, 7 + __ = 11)	› add and subtract numbers up to 100 without renaming (e.g., 53 + 21, 55 – 32)	› add and subtract numbers up to at least 100 (e.g., 43 – 28, 37 + 18)
		› explore addition facts up to 10 and their corresponding subtraction facts (families of facts), including doubles and halves	› recall addition facts up to 10, and explore addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves	› recall addition facts up to 20 and their corresponding subtraction facts (families of facts), including doubles and halves

Teaching considerations
Investigate and **represent** the partitioning of numbers using appropriate materials for the year level – for example: › multilink cubes, bead strings, 10s frames, and counters, at 6 months and year 1 › a Slavonic abacus, ice-block sticks, and PV money, at year 2 › PV money and PV blocks, at year 3. **Connect** students' subitising with pattern understanding (at 6 months and year 1) and known groupings and facts (at years 2–3). **Explain** and discuss how to systematically record the partitioning of numbers (e.g., using partitioning diagrams, tables, vertically-listed equations).
Explain and spend time developing the concepts of: › estimation, using the language of 'about', 'more or less', and 'close to' › rounding, using 100s boards and number lines marked with the multiples of 10 or 100, progressing to unmarked number lines at year 3. Have students **investigate** and **connect** practical estimation situations that involve quantities and measures (e.g., the number of balls in a box, the number of steps to the door, the length of a piece of string).
Explain and discuss addition and subtraction using **representations**, including: › discrete materials (counters, blocks, context items), 10s frames, and number lines (at 6 months and year 1) › bundles of sticks, number disks, and number lines (at year 2) › PV materials (PV money, blocks, and discs) and number lines (at year 3). **Connect** symbols and equations with problems, using correct vocabulary (e.g., 'add', 'join', and 'plus' for addition). Have students practise decoding and solving word problems. At year 3, **explain and connect** horizontal equations and the vertical-column method for addition and subtraction. Demonstrate making estimates or mental calculations by **connecting** to place value, partitioning, and known facts. Use a range of problem types (e.g., result, change, start-unknown). Use worked examples and think-alouds to **explain** the most efficient approaches when solving problems. Have students **investigate** and **generalise** adding 0 to or subtracting 0 from a number (at year 1) and applying the commutative property of addition (e.g., 5 + 4 = 4 + 5).
Use materials to **investigate** addition and subtraction facts (e.g., counters, 10s frames, an abacus, multilink cubes), and use part-whole diagrams to develop subtraction facts and **connect** to addition facts. **Explain** how to record equations and families of facts, **connecting** with the language for each operation. Provide a range of tasks to consolidate learning and develop fluency (e.g., physical and digital games, using families of facts and, at year 3, table grids).

	During the first 6 months *Informed by prior learning, teach students to:*	During the first year *Informed by prior learning, teach students to:*	During the second year *Informed by prior learning, teach students to:*	During the third year *Informed by prior learning, teach students to:*
Operations			› identify the relationship between skip counting and multiplication facts for 2s, 5s, and 10s	› recall multiplication and corresponding division facts for 2s, 3s, 5s, and 10s
		› multiply and divide using equal grouping or counting	› multiply and divide using equal grouping or skip counting (e.g., in 2s, 5s, and 10s)	› multiply a one- or two-digit number by a one-digit number, using skip counting or known facts (e.g., 4 × 6, 2 × 23)
				› divide whole numbers by a one-digit divisor with no remainders, using grouping (e.g., 24 ÷ 3, 32 ÷ 4)
Rational numbers		› identify and represent halves and quarters as fractions of sets and regions, using equal parts of the whole	› identify, read, write (using symbols and words), and represent halves, quarters, and eighths as fractions of sets and regions, using equal parts of the whole	› identify, read, write, and represent halves, thirds, quarters, fifths, sixths, and eighths as fractions of sets and regions, using equal parts of the whole and by positioning on a number line
			› directly compare two fractions involving halves, quarters, and eighths	› compare and order fractions involving halves, quarters, and eighths and identify when two fractions are equivalent

Teaching considerations
Use a range of materials to **represent** skip counting and multiplication and division facts (e.g., 100s boards, choral counting, games, number lines, a Slavonic abacus, families of facts, and, at year 3, table grids). Provide a range of tasks to consolidate learning and develop fluency.
Represent multiplication and division problems using discrete materials, pictures, diagrams, symbols, number lines, words, equations, digital tools, and, at year 3, arrays, PV materials, and bar models. Use correct mathematical language when discussing multiplication and division (e.g., multiply, groups of, sets of, rows of, equal groups, divide, share equally). Have students practise decoding and solving word problems. **Connect** with subitising and addition and subtraction concepts when demonstrating solving multiplication and division problems. **Explain** and **represent** division as a sharing problem (e.g., "Share 12 marbles equally among 3 friends") or a grouping problem (e.g., "You have 12 marbles. How many groups of 3 marbles can you make?"). Use worked examples and think-alouds to **explain** the most efficient approaches when solving multiplication and division problems. **Investigate** and **generalise** multiplying a number by 0 or 1, dividing a number by 1, dividing a number by itself, and why we cannot divide by 0 (e.g., by trying to solve 0 × __ = 5). At year 3, **explain** and use the multiplicative identity (e.g., 5 × 1 = 5, 4 ÷ 1 = 4) and commutative property (e.g., 3 × 4 = 4 × 3). Demonstrate making estimates or mental calculations by **connecting** to place value, partitioning, and known facts.
Represent fractions using a range of materials – continuous (bar models, number lines), discrete (sets of objects), and digital. **Explain** and reinforce that when fractions are represented symbolically: › the denominator is the bottom number and shows how many pieces a whole has been equally split into › the numerator is the top number and shows how many of those parts the fraction represents. Have students practise saying, reading, and writing fractions in words and symbols. **Explain** how to fold paper strips to create fractions of one whole. Label the parts using words and symbols, and use them to create a fraction wall for comparing and ordering fractions. **Explain** that a fraction is a number that can be placed on a number line.

	During the first 6 months *Informed by prior learning, teach students to:*	During the first year *Informed by prior learning, teach students to:*	During the second year *Informed by prior learning, teach students to:*	During the third year *Informed by prior learning, teach students to:*
Rational numbers		› find a half or quarter of a set using equal sharing and grouping.	› find a half and quarter of a set by identifying groups and patterns (rather than sharing by ones), and identify the whole set or shape when given a half or quarter	› find a unit fraction of a whole number (e.g., $\frac{1}{3}$ of 15), and identify the whole set or amount when given a unit fraction (e.g., "$\frac{1}{4}$ of the set is 3, what is the whole set?")
				› add and subtract unit fractions with the same denominator (e.g., $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$)
Financial mathematics			› recognise and order New Zealand denominations up to $20 according to their value, make groups of 'like' denominations, and calculate their value.	› make amounts of money using one- and two-dollar coins and 5-, 10-, 20-, 50-, and 100-dollar notes.

Teaching considerations
Investigate a range of practical situations using a range of **representations**, including materials, drawings and diagrams, and digital tools (e.g., discrete objects, bar models, paper strips for partitioning). Make **connections** between: › symbols, words, and pictures › counting, subitising patterns and known groupings, and skip counting to solve problems (at years 1–2) › skip counting and using known addition and multiplication facts to solve problems (at year 3). Use mathematical language to develop an understanding of fractions (e.g., numerator, denominator, shared equally, divide, partition, equal parts).
Investigate adding and subtracting fractions within familiar contexts (e.g., cutting apples into eighths or partitioning paper strips into six equal parts, and then **representing** addition and subtraction with these materials). **Connect representations**, including symbols and equations, to drawings and materials (e.g., fraction walls, paper fraction strips), and show them on a number line.
Have students use play money (coins and notes) to **represent** practical financial situations. At year 2, compare only notes with notes or cents with cents, not a mixture of them. At year 2, **investigate** appropriate financial situations that involve both saving and spending. **Connect** to place value, addition and subtraction, and skip counting when calculating amounts.

Algebra

	During the first 6 months *Informed by prior learning, teach students to:*	During the first year *Informed by prior learning, teach students to:*	During the second year *Informed by prior learning, teach students to:*	During the third year *Informed by prior learning, teach students to:*	Teaching considerations
Equations and relationships		› solve true or false number sentences and open number sentences involving addition and subtraction of one-digit numbers, using an understanding of the equal sign (e.g., 2 + 5 = 3 + __, 7 – 5 = 6 – 4 (T or F?)	› solve true or false number sentences and open number sentences involving addition and subtraction of one- and two-digit numbers, using an understanding of the equal sign (e.g., 18 + __ = 17 + 6, 17 = 25 (T or F?)	› solve true or false number sentences and open number sentences involving addition and subtraction, using an understanding of the equal sign	**Represent** the equal sign as the 'same as' to demonstrate it is a symbol of equivalence. **Investigate** number sentences using **representations** such as: › 10s frames and discrete materials (at years 1–2) › word problems with comparisons (at year 3). At years 2–3, solve number sentences that have numbers beyond what students are using in operations, so that the emphasis is on the equal relationship, not operating.
	› copy, continue, create, and describe a repeating pattern with two elements.	› copy, continue, create, and describe a repeating pattern with three elements, and identify missing elements in a pattern	› recognise and describe the unit of repeat in a repeating pattern, and use it to predict further elements using the ordinal position	› recognise, continue, and create repeating and growing patterns, and describe a rule to explain a pattern	**Investigate** repeating and growing patterns in a range of contexts (e.g., cultural patterns, patterns in the local environment and on everyday objects). Use materials, sound, movement, and digital tools to **represent** and continue repeating and growing patterns. At years 2–3, demonstrate recording the pattern in a table. Form **generalisations** when students notice that repeating patterns constructed in different ways are similar (e.g., 'red, blue, red, blue' and 'hop, jump, hop, jump' are ABAB patterns). Help students to notice the similarities and differences between patterns by recording them. With students at year 2, **generalise** by using the unit of repeat and ordinal position to identify further elements in a pattern. Use mathematical language and sentence starters to support students to **explain** and **justify** how a pattern is repeating or growing and to predict further terms.
Algorithmic thinking		› follow step-by-step instructions to complete a simple task.	› follow and give step-by-step instructions for a simple task, identifying and correcting errors as the instructions are followed.	› create and use a set of precise, step-by-step instructions for carrying out a familiar routine or task.	**Represent** step-by-step instructions using drawings, words, flow diagrams, and verbal instructions that form a sequence. With students, **investigate** sorting unfamiliar and familiar objects according to a set of instructions, directing a person or object (e.g., through an obstacle course or maze), and following and creating a set of pictorial instructions. **Explain**, **justify**, and show how a set of instructions is complete or incomplete, using think-alouds and prompts. **Connect** a series of events from a story, narrative, or daily timetable with statements in Number, Algebra, Measurement, and Geometry.

Measurement

	During the first 6 months *Informed by prior learning, teach students to:*	**During the first year** *Informed by prior learning, teach students to:*	**During the second year** *Informed by prior learning, teach students to:*	**During the third year** *Informed by prior learning, teach students to:*	**Teaching considerations**
Measuring			› estimate and use an informal unit repeatedly to measure the length, mass (weight), volume, or capacity of an object	› estimate and then reliably measure length, capacity, and mass (weight) using whole-number metric units (e.g., from tools with labelled markings)	**Explain** estimation, using the language of 'about', 'more or less', and 'close to' to help students reflect on what the quantity or measure might be. **Investigate** practical estimating and measuring situations, using appropriate measuring tools (e.g., at year 2, balance scales, capacity containers, informal units; at year 3, rulers, measuring jugs and cups, scales). At year 3, **explain** how to construct and use measurement devices, particularly rulers, measurement containers, and balance scales. Demonstrate how to accurately measure length in centimetres, mass (weight) in grams, and capacity in millilitres (at year 3).
	› directly compare two objects by an attribute (e.g., length, mass (weight), capacity)	› compare the length, mass (weight), volume, or capacity of objects directly or indirectly (e.g., by comparing each of them with another object, used repeatedly)	› compare and order several objects using informal units of length, mass (weight), volume, or capacity	› compare and order objects using metric units of length, mass (weight), or capacity	**Investigate** practical measuring situations to compare and order objects – for example: › which is longer or shorter, is heaver or lighter, or holds more or less (at 6 months) › comparing and ordering up to three objects (at year 1) › explaining how identical informal units need to be used when measuring (at year 2) › using tools like rulers, measurement containers, and scales (at year 3). Use mathematical language to **explain** and **justify** comparative measurement attributes (e.g., long and short; heavy, heavier, and heaviest; the same as; full and empty; more and less; wide, wider, and widest). Include descriptive te reo Māori that makes the properties of objects and shapes clear.
			› turn, and describe how far an object or person has turned, using full, half, and quarter turns as benchmarks	› turn, and describe how far an object or person has turned, using full, half, quarter, and three-quarter turns as benchmarks	**Investigate** and **explain** situations involving angles as 'how far an object or person has turned.' Have students turn physical objects and themselves. **Connect** turns with fractions (e.g., half, a quarter, three quarters).
	› connect days of the week to familiar events and daily routines (e.g., the class timetable).	› identify how the passing of time is measured in years, months, weeks, days, hours › name and order the days of the week, and sequence events in a day using everyday language of time	› name and order the months and seasons, and describe the duration of familiar events using months, weeks, days, and hours	› identify the duration of events using years, months, weeks, days, hours, minutes, and seconds	Use visual **representations** to support the sequencing of events (e.g., pictorial daily timetables, calendars, day-and-month cards). Explore estimating the duration of everyday events using minutes and seconds (e.g., "How long is it until the bell rings?"). Practise recalling a sequence of events in the past and predicting future events. Use mathematical language to **explain** and **justify** comparisons of duration and points in time (e.g., before, after, soon, later, next, today, tomorrow, yesterday, 1st, 2nd, 3rd). **Investigate** using a calendar to work out the number of days, weeks, or months until important events (e.g., the number of days until Matariki, the number of weeks until the end of term). Explore informal ways of measuring short periods of time to identify which events last longer.

	During the first 6 months *Informed by prior learning, teach students to:*	**During the first year** *Informed by prior learning, teach students to:*	**During the second year** *Informed by prior learning, teach students to:*	**During the third year** *Informed by prior learning, teach students to:*
Measuring		› tell the time to the hour using the language of 'o'clock'.	› tell the time to the hour and half-hour, using the language of 'past' and 'o'clock'	› tell the time to the hour, half-hour, and quarter past and quarter to the hour
Perimeter, area, and volume			› visualise, estimate, and measure the perimeter and area of 2D shapes, using informal units.	› visualise, estimate, and measure: – the perimeter of polygons using metric units – the area of 2D shapes using squares of identical size – the volume of rectangular prisms (cuboids) by filling them with identical 3D blocks.

Teaching considerations

Measuring

Use digital and analogue clocks to have students practise telling the time. **Connect** using visual representations on an analogue clock to skip counting in 5s and fractions (a half and quarter).

Connect the 'structure' of duration (minutes, hours, days) to our measures of time ("There are 30 minutes in half an hour, 60 minutes in an hour").

Identify and **investigate** the specific times of daily events and activities in and out of school.

Perimeter, area, and volume

Explain and demonstrate that:

- › perimeter is the distance around the boundary of a 2D shape
- › area is the size of the surface of a 2D shape, or how many squares cover the surface
- › volume is the amount of 3D space a shape takes up, or how many cubes fill the shape.

Investigate familiar practical situations involving perimeter, area, and volume.

Use think-alouds to demonstrate the use of visualising to identify the appropriate attribute for a measurement task and to imagine the number of units required.

Explain the importance of using the same unit when measuring, and that there should be no gaps or overlaps around the outside (perimeter) and inside (area) of 2D shapes and in filled 3D shapes (volume).

Geometry

	During the first 6 months *Informed by prior learning, teach students to:*	**During the first year** *Informed by prior learning, teach students to:*	**During the second year** *Informed by prior learning, teach students to:*	**During the third year** *Informed by prior learning, teach students to:*	**Teaching considerations**
Shapes	› identify, sort by one feature, and describe familiar 2D shapes	› identify, describe, and sort familiar 2D and 3D shapes presented in different orientations, including triangles, circles, rectangles (including squares), cubes, cylinders, and spheres	› identify, describe, and sort 2D and 3D shapes, including ovals, semicircles, polygons (e.g., hexagons, pentagons), rectangular prisms (cuboids), pyramids, hemispheres, and cones, using the attributes of shapes	› visualise, identify, compare, and sort 2D and 3D shapes, using the attributes of shapes	Make available a range of 2D and 3D shapes, including tactile shapes and materials (e.g., playdough, pipe cleaners), pictures, diagrams, and digital tools. **Investigate** 2D and 3D shapes in the environment. Use everyday language and mathematical language (including te reo Māori) to **explain** and **justify** the describing and sorting of shapes (e.g., size, corners, colour, texture, sides, angles, faces, edges, vertices, triangle/tapatoru, square/tapawhā rite, same/ōrite, different/rerekē). Use **generalisations** made by students to clarify and extend understanding (e.g., "Polygons have straight sides", "2D shapes can be identified on 3D shapes").
				› identify right angles in shapes and objects	
Spatial reasoning	› compose by trial and error a target shape using smaller shapes, and decompose a shape into smaller shapes	› anticipate which smaller shapes might be used to compose a target shape, and then check by making the shape	› anticipate which smaller shapes might be used to compose and decompose a target shape, and then check by making the shape	› compose and decompose 2D shapes using the attributes of shapes (e.g., lines of symmetry), other shapes, side lengths, and angles	Make available a range of materials to compose and decompose 2D shapes (e.g., pattern blocks, attribute shapes, paper shapes, playdough, tangrams). Use think-alouds to demonstrate anticipating how small shapes can fit into or make a new shape. Use as target shapes: › shapes partitioned into smaller parts (at 6 months) › continuous whole shapes with no partitions (at years 1–3).
		› flip, slide, and turn 2D shapes to make a pattern	› recognise lines of symmetry in patterns or pictures, and create or complete symmetrical pictures or patterns	› predict the result of a one-step transformation (reflection, translation, or rotation) on 2D shapes	**Connect** the informal vocabulary of flip, slide, and turn with the formal vocabulary of reflect, translate, and rotate. **Investigate** practical situations (e.g., making art, paper folding, checking symmetry with mirrors) and a range of artefacts and patterns.
Pathways	› follow instructions to move to a familiar location or locate an object.	› follow and give instructions to move to a familiar location or locate an object	› follow and give instructions to move people or objects to a different location, using direction, distances (e.g., number of steps), and half and quarter turns	› follow and create a sequence of step-by-step instructions (an algorithm) for moving people or objects to a different location	**Investigate** ways of moving to different locations within the classroom and in other parts of the school, using simple maps at year 3. Use picture books that emphasise positional language and movement (e.g., *Scatter Cat, Bears in the Night, We're Going on a Moa Hunt*). Use spatial language and talk frames to support giving and following instructions (e.g., near, far, next to, beside, on top, under, over, down, up, left, right, turn). Make **connections** between: › estimating distance and bodily measures (e.g., the number of steps to the door) › half and quarter turns and fractions › following or creating instructions and algorithmic thinking.
		› use pictures, diagrams, or stories to describe the positions of objects and places.	› interpret diagrams to describe the positions of objects and places in relation to other objects and places.	› interpret, draw, and use simple maps to locate objects and places relative to other objects and places.	

Statistics

	During the first 6 months *Informed by prior learning, teach students to:*	**During the first year** *Informed by prior learning, teach students to:*	**During the second year** *Informed by prior learning, teach students to:*	**During the third year** *Informed by prior learning, teach students to:*
Problem		› pose a summary investigative question about a group for which the data will have categorical variables (e.g., colour, brand), and anticipate what the data might show	› pose a summary investigative question about a group for which the data will have categorical variables, and anticipate what the data might show (e.g., which outcomes might be more frequent than others)	› pose a summary investigative question about an everyday situation, using categorical data and discrete numerical (whole number) data, identify the variable and group of interest, and anticipate what the data might show
Plan		› plan to collect data by making observations or questioning others, and discuss how the data-gathering process might affect people	› plan survey and data-collection questions for collecting data, identify who and what the data will measure, and discuss how the data-gathering process might affect people	
Data		› collect categorical data for one variable	› collect categorical data for more than one variable	› collect, record, and sort data, or use secondary data sources provided by someone else
Analysis		› create and make statements about data visualisations (e.g., pictures, graphs, dot plots) for the categorical data, giving the frequency for each category	› create and make statements about data visualisations (e.g., pictures, graphs, dot plots) for the categorical data, comparing the frequencies of categories	› create and make statements about data visualisations (e.g., pictures, graphs, dot plots, bar graphs) for the categorical and discrete numerical data
Conclusion		› choose from given options the statements that best answer the investigative question		› choose from given options the statements that best answer the investigative question, reflect on findings, and compare them with anticipated outcomes

Teaching considerations
Show, with student input, how to: › pose summary **investigative** questions about an area of interest › identify the variable and group of interest in investigative questions.
Pose, with student input, survey and data-collection questions that will be used to collect the data required for the investigative question. **Explain** the distinction between primary and secondary data and the challenges that come with sensitive topics or questions. **Investigate** how survey questions and the words within survey questions can be interpreted differently by different people.
Represent data using data cards, recording sheets, and tally tables. Use data cards that **represent** multiple variables about an individual. Explore investigative questions using secondary data sources.
Show creating and describing data visualisations, transitioning from data cards to dot plots to bar graphs. **Represent** data using data cards and picture graphs (for years 1–3), frequency tables and dot plots (for years 2–3), and bar graphs (for year 3). Have students practise using 'I notice' statements that include the variable name and context when describing data visualisations. **Explain** and demonstrate 'reading the data' and 'reading between the data'. **Explain** how to describe features of data visualisations (e.g., frequency, the least/most frequent category, modes or modal groups, highest and lowest values).
Show, with student input, how to: › choose the best descriptive statements that answer an investigative question › collate, **explain**, and **justify** their findings to others.

	During the first 6 months *Informed by prior learning, teach students to:*	During the first year *Informed by prior learning, teach students to:*	During the second year *Informed by prior learning, teach students to:*	During the third year *Informed by prior learning, teach students to:*
Statistical literacy		› agree or disagree with others' statements about simple data visualisations (e.g., pictures, graphs, dot plots).	› match statements made by others with features in simple data visualisations, and agree or disagree with the statements.	› identify relevant features in others' data visualisations, connect these to descriptive statements, agree or disagree with the statements, and suggest improvements to them.

Teaching considerations

Show, with student input, how to:

› read and understand claims made by others and identify corresponding features in data visualisations

› **explain** agreements or disagreements with a claim made by others.

Probability

	During the first 6 months *Informed by prior learning, teach students to:*	During the first year *Informed by prior learning, teach students to:*	During the second year *Informed by prior learning, teach students to:*	During the third year *Informed by prior learning, teach students to:*
Probability investigations		› engage in stories or games that involve chance-based situations and: – decide if something will happen, won't happen, or might happen – identify possible and impossible outcomes (e.g., for what might happen next).	› engage in chance-based investigations about games and everyday situations to: – anticipate and then identify possible outcomes – collect and record data – create data visualisations for frequencies of possible outcomes (e.g., lists, pictures, graphs) – describe what these visualisations show – answer the investigative question – notice variations in outcomes (e.g., how often each of the numbers on a dice come up)	
Critical thinking in probability			› agree or disagree with the statements made by others about chance-based situations.	› explain and question statements about chance-based situations, with reference to data.

Teaching considerations

Probability investigations:

Investigate probability by playing games of chance using physical objects (e.g., dice, coins, spinners, pulling things out of a hat).

Explain and show how to:

› list possible outcomes

› visualise frequencies of outcomes

› use the vocabulary that indicates the relative order of probabilities from impossible to certain (i.e., impossible, unlikely, possible, likely, certain).

Critical thinking in probability:

Show, with student input, how to:

› read and understand claims made by others about chance situations

› match statements with the relevant chance situation being described

› **explain** and **justify** why they believe a statement is true or not.

The language of mathematics and statistics: Phase 1

	At 6 months *Students will know the following words:*	Year 1 *Students will know the following new words:*	Year 2 *Students will know the following new words:*	Year 3 *Students will know the following new words:*
Number	› add, plus, join › altogether › biggest, smallest › combine, separate › count › group › how many › in between › more, less › next, before, after › ordinal (1st, 2nd, 3rd etc.) › takeaway, minus	› count on, count back › digit › double, halve › equal group › equal part › fair share › forwards, backwards › fraction › half, quarter › odd, even › partition › set › share › skip count › subtract › sum, difference › whole set	› cent, coin, dollar, note › denominator › eighth › estimate, estimation › money › multiply, divide › numerator › place value › quantity, amount › regroup	› operation › round › third, fifth, sixth › unit fraction
Algebra	› continue › copy › next › pattern › repeat	› changed, unchanged › element › equal, equivalent › equation › number sentence › repeating pattern › true, false › unit of repeat › zero	› error › predict	› complete, incomplete › growing pattern › rule › sequence › term
Measurement	› comparative words (long, taller, heaviest etc.) › full, empty › heavy, light › height › length › measure, weigh › same as › short, tall, wide, large, small, big	› capacity › day, week, month, year › days of the week › distance › earlier, later › hour › morning, afternoon, evening › o'clock › starting point, end point › weight	› area › full turn, half turn, quarter turn › half past › months of the year › perimeter › seasons of the year › surface › width	› gram › litre, millilitre › measuring jug or cup › metre, centimetre › metric › minute, second › quarter past, quarter to › ruler › three-quarter turn › unit › volume › weighing scale, balance scale

	At 6 months *Students will know the following words:*	Year 1 *Students will know the following new words:*	Year 2 *Students will know the following new words:*	Year 3 *Students will know the following new words:*
Geometry	› flip › positional language (next to, above, below, under, up, down, on top of, inside etc.) › side, corner › size (big, small, long, short) › square, triangle, circle › straight, curved, round › turn	› 2D shape › 3D or solid shape › cube, cylinder, sphere › edge, face › slide › rectangle	› direction › left, right › oval, semicircle, polygon (hexagon, pentagon), rectangular prism (cuboid), pyramid, hemisphere, cone › position › symmetry, line of symmetry › vertex	› location › quadrilateral › reflect, reflection › right angle › rotate, rotation › transform, transformation › translate, translation
Statistics		› data › dot plot › information › most, least › picture graph › survey › tally	› category › graph › notice › outcome › statement › table › title	› bar graph › claim › finding › frequency › variable
Probability		› chance › possible, impossible › will happen, won't happen, might happen	› agree, disagree › anticipate › certain, uncertain › likely, unlikely › list	› probability

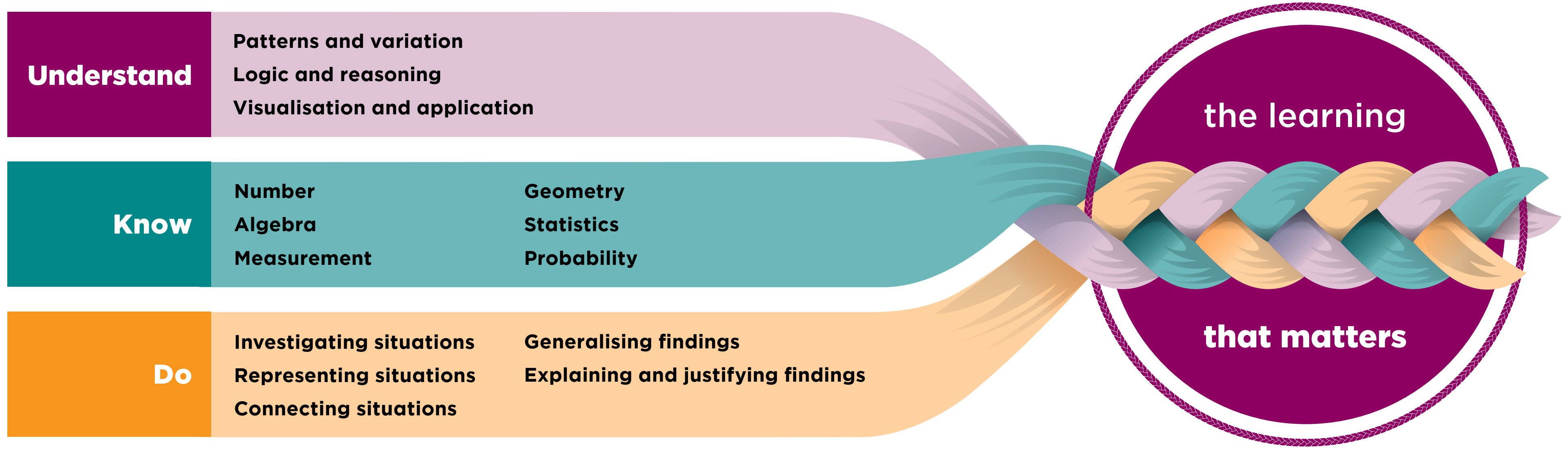
Phase 2
Years 4–6

Progress outcome by the end of year 6

Expanding horizons of knowledge, and collaborating
Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi

The critical focus of phase 2 is for all students to expand their horizons of knowledge and their collaboration with others. Students use a variety of representations to model number operations and to solve word problems. They connect and extend their reasoning about whole numbers to fractions and decimals, and they visualise and classify angles, using benchmarks to justify their classifications. Students also apply their understanding of number operations to reasoning about perimeter and area and to investigating variations in patterns, shapes, and data.

The phase 2 progress outcome describes the understanding, knowledge, and processes that students have multiple opportunities to develop over the phase.



The phase 2 progress outcome is found on the following two pages.

Understand

As students build knowledge through their use of the mathematical and statistical processes, they develop their understanding of the following.

Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical-thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

Know

Number | Mātauranga tau

By the end of this phase, students know that in our number system each place value is a power of 10, and this continues infinitely. To the right, the system continues past the ones column to create decimals (tenths, hundredths, thousandths); the decimal point marks the column immediately to the right as the tenths column. Estimation and rounding support checking the reasonableness of solutions of operations involving whole numbers, fractions, and decimals. Students know that to evaluate expressions that have more than one operation, operations inside brackets (i.e., grouped together) are done first. If there are multiplication and division, these are then done in left-to-right order; finally, addition and subtraction are also done in left-to-right order. Division can be partitive (the number of shares is known) or quotitive (the size of the shares is known).

Students come to recognise the properties of number operations. The additive identity is 0 (e.g., 3 + 0 = 3) and the multiplicative identity is 1 (e.g., 5 × 1 = 5). The commutative property (e.g., 3 × 5 = 5 × 3) and associative property (e.g., 3 × (4 × 6) = (3 × 4) × 6) apply to addition and multiplication but not to subtraction and division. The distributive property (e.g., 2 × (6 – 4) = 2 × 6 – 2 × 4) applies to multiplication over addition and subtraction.

Students also come to know that fractions can result from one number divided by another (the quotient), operate on quantities, and be larger than 1. Improper fractions can also be written as a mixed number, represented as a whole number and a fraction, combined with a hidden addition. In simplified fractions, the numerator and denominator have no common factors; if the denominator of a simplified fraction is 1, then it can be written as a whole number. Decimals are fractions that have powers of 10 as their denominators, and they can be written as numbers using a decimal point. A percentage is a fraction with a denominator of 100.

Algebra | Taurangi

By the end of this phase, students know that the equal (=) and inequality (<, >) signs show relationships, and that applying the same operation to both sides of an equation preserves the balance of the equation. Students know that in a pattern, the relationship between the ordinal position and its corresponding element can be used for finding the pattern rule. Any element can be found by knowing its position, and any position can be found from its corresponding element. Tables and XY graphs provide a way of organising the positions and elements of a pattern to reveal relationships or rules. An algorithm is an ordered list of instructions for solving a problem.

Measurement | Ine

By the end of this phase, students know that, like our place-value number system, the metric measurement system is based on powers of 10 and that appropriate metric units are used to quantify length, area, volume, capacity, mass (weight), and temperature. Measurements can include whole units and parts of units. Different measurement tools and scales use different-sized units, and the unit must be recorded with the amount. Duration is the amount of time it takes for an event to occur. Angles are a measure of turn and can use the unit of degrees.

Geometry | Āhuahanga

By the end of this phase, students know that two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between shapes. Shapes can be transformed by rotation, reflection, translation, and resizing (when they are enlarged or reduced). Lines of symmetry can be horizontal, vertical, and diagonal. Three-dimensional shapes can be composed of connected two- or three-dimensional shapes. Students also know that position can be described using known environmental features and elements from the natural world. Maps can use grid references to specify the position of locations, scales to show distances, and connections to show pathways.

Statistics | Tauanga

By the end of this phase, students know that data about people and the natural world must be collected, used, and stored carefully. The statistical enquiry cycle (PPDAC) can be used in summary, comparison, and time-series investigations. A comparison investigation compares similarities and differences for a variable across two or more groups, and a time-series investigation considers how a variable changes over time. Numerical variables can be counted or measured; discrete numerical variables are counted, continuous numerical variables are measured. A conjecture or assertion involves thinking about what data will show before it is collected or analysed. Data is not always accurately recorded; it needs to be checked for errors and may need correcting. Alternative data visualisations for the same data can lead to different insights.

Probability | Tūponotanga

By the end of this phase, students know that the statistical enquiry cycle (PPDAC) can be used for chance-based investigations. Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain). They can be used to describe situations that involve uncertainty and help make decisions. In a chance-based investigation, the probability of an outcome is the relative frequency of the outcome in a probability experiment (the probability estimate). If outcomes are believed to be equally likely, the probability of an outcome is the number of times the outcome occurs divided by the total number of outcomes, where all possible outcomes can be listed (the theoretical probability).

Do

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it step by step. They can identify relevant prior knowledge, conditions, and relationships to support the investigation. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adopt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate objects and ideas.

Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They also make connections with ideas in other learning areas and with familiar contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively, based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, evaluating their logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing and building on ideas with others. They present reasoned explanations and arguments for an idea, solution, or process.

Phase 2 Years 4–6

Teaching sequence

Expanding horizons of knowledge, and collaborating
Te whakawhānui i ngā pae o te mātauranga, me te mahi tahi

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the second phase of learning at school.

Throughout phase 2, encourage students to see themselves as capable, confident, and competent mathematics and statistics thinkers whose ideas are valued, who treat mistakes as part of the learning process, and whose abilities in mathematics and statistics will develop over time with consistent effort. Confidence is built through experiencing success and developing competence and understanding. Over phase 2, students collaborate with others to expand their knowledge and understanding. Support this by working with the class to establish expectations and responsibilities for working together as peers, sharing thinking, and agreeing or disagreeing about mathematical and statistical learning.

Continuously monitor students' cognitive load, reasoning, questions, and use of representations, and respond quickly to address any issues and misconceptions. Ensure teaching builds on what students already understand, know, and can do.

Explicit teaching

- Use warm-up routines as a form of active recall that connects back to prior learning (e.g., quick challenges, curly questions, games). Plan for students to develop fluency through practice, using a range of approaches.
- Use worked examples and break down new learning into clearly explained, manageable steps. Use mathematical and statistical symbols and notation conventions, explaining them and how they work. Teach conceptual understanding of number operations and efficient written and mental methods for them.
- Connect mathematical and statistical learning within and across contexts. Teach connected procedures and concepts together (e.g., multiplication and division with area and volume). Make connections explicit by highlighting concepts students have applied in other learning areas.
- Plan ways for students to consolidate their mathematical and statistical learning. Use prompts, questions, and situations that incorporate previously taught concepts and procedures, to help students retrieve and apply them. Highlight connections with new learning.

Positive relationships with mathematics and statistics

- Encourage curiosity through exploring mathematics and statistics in, for example, history, games, art, and puzzles.
- Highlight to the class the mathematical thinking and approaches of individuals or groups. Display drawings or photos of students' representations and workings throughout the learning process, and use these to start conversations about mathematical and statistical learning and progress between students and with families.

Rich tasks

- Plan to explore rich mathematical and statistical situations and contextual tasks that are useful and meaningful to the class or community.
- Design tasks that use different contexts or combinations of operations to encourage students to apply their reasoning and knowledge to other types of problems (e.g., using decimals in measurement situations).
- Encourage students to generalise by using questions such as "If I change this, what happens to that?" and "Is there another way to show this?"
- Teach problem-solving and investigation strategies. Support students to read and make sense of a problem – through drawing, using materials, or trying some numbers – and then to identify relevant knowledge, plan how to solve the problem in a sequence of steps, take action to apply their plan (recording calculations with meaningful explanations), and check their findings.
- Give students opportunities to notice and wonder about patterns, structures, and relationships and make statements about them.

Communication in mathematics and statistics

- Plan for students to actively listen to, reflect, and build on each other's thinking and learning. Use discourse-based tools and a range of open questions to facilitate productive discussions. Over the phase, encourage students to use evidence and examples to justify their claims and findings.
- Select appropriate representations to show working and reasoning. Over the phase, move students towards using pictures, diagrams, and mathematical notation such as equations and inequalities. Teach students which representations are most effective for showing different types of information (e.g., number lines are important for representing operations, differences, the comparative size of numbers, and rounding conventions).
- Prompt students to visualise and share their thinking about quantities, patterns, shapes, measurements, and data. Support students to visualise by estimating the number of items in a group, using rounding or known benchmarks to make estimations, and by noticing and responding to how a shape has been rotated or reflected, or is composed of other shapes.
- Use mathematical and statistical language. Demonstrate the use of correct vocabulary that connects to the learning purpose or problem. Ask students to use correct vocabulary and to explain their findings and reasoning. In doing so, draw on students' first and heritage languages, so that they can use their languages as a resource to connect their thinking and learning.
- Dedicate time for students to record learning (e.g., in their mathematics and statistics book). Support them to organise their ideas clearly, using words, mathematical notation, and a range of representations. Provide opportunities for them to consider their goals and to reflect on their learning.

Number

	During year 4 *Informed by prior learning, teach students to:*	During year 5 *Informed by prior learning, teach students to:*	During year 6 *Informed by prior learning, teach students to:*	Teaching considerations
Number structure	› skip count from any multiple of 100, forwards or backwards in 25s and 50s			**Investigate** patterns in multiples, using 100s boards or 1,000s books. Record choral counting on the board, and ask students to **explain** patterns and make **generalisations** or conjectures.
	› identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure	› identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure	› identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure	Use marked number lines to order and compare numbers and place-value (PV) houses and materials to write and **represent** numbers, using base 10 structure. Support students to: › practise saying, reading, and writing given numbers, including large numbers, using PV houses › use PV houses to **generalise** that multiplying by 10 moves each digit in a number one place to the left, and dividing by 10 moves each digit one place to the right.
		› identify factors of numbers up to 100	› identify square numbers and factors of numbers up to 125	**Represent** factors of numbers using arrays or ordered lists of factor pairs. Use multiplication charts to **investigate** factors, multiples, and square numbers. **Connect** to students' understanding of a square to **explain** and **represent** a square number and multiplication facts involving the same two numbers.
Operations	› use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations	› use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations	› use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations	**Explain** how to round numbers to an appropriate value to make an estimate for a calculation. **Explain** reasoning using estimation language such as 'about', 'more or less', and 'close to'. **Connect** rounding with: › known benchmarks (e.g., doubles, halves, multiples of 10), to make estimations and check calculations › rounding to an appropriate unit in measurement situations. Use number lines to support rounding, **explaining** how to find the midpoint between two numbers. **Explain** and **justify** findings by connecting to estimates and other checking methods. Use families of facts to show the **connection** between factors and multiples. **Explain** how to use families of facts to 'work backwards' (e.g., 7 × 8 = 56, so 56 ÷ 8 = 7).
	› round whole numbers to the nearest thousand, hundred, or ten	› round whole numbers to the nearest ten thousand, thousand, hundred, or ten, and round tenths to the nearest whole number	› round whole numbers to a specified power of 10, and round tenths and hundredths to the nearest whole number or one decimal place	
	› add and subtract two- and three-digit numbers	› add and subtract whole numbers up to 10,000	› add and subtract any whole numbers	**Explain** and **represent** addition and subtraction using materials such as PV materials, number lines, and number discs. **Explain** and **connect**: › the horizontal method and the vertical-column method of addition or subtraction › making estimates or mental calculations using place value, partitioning, and known facts. Use worked examples and a range of problem types (e.g., result, change, start-unknown), using think-alouds to **explain** the most efficient approaches. Have students practise decoding and solving word problems, **representing** them as equations.

	During year 4 ***Informed by prior learning, teach students to:***	**During year 5** ***Informed by prior learning, teach students to:***	**During year 6** ***Informed by prior learning, teach students to:***	**Teaching considerations**
Operations	› recall multiplication and corresponding division facts for 4s and 6s	› recall multiplication facts for 7s, 8s, and 9s and corresponding division facts	› recall multiplication facts to at least 10 × 10 and corresponding division facts	Provide a range of tasks for students to practise and develop fluency in new and previously learned multiplication and division facts (e.g., families of facts, multiplication table grids, arrays, games). **Investigate** patterns in the multiples of times tables and to **generalise** multiplication problems beyond recalled facts by looking for patterns.
	› multiply a two-digit by one-digit number and two one-digit whole numbers (e.g.,23 × 5, 7 × 8)	› multiply a three-digit by one-digit number and two two-digit whole numbers (e.g., 245 × 6, 34 × 83)	› multiply multi-digit whole numbers (e.g., 54 × 112)	At year 4: › **connect** multiplication with skip counting using jumps on a number line or arrays › **represent** division using diagrams and equal sharing, connecting with known families of facts › **generalise** the distributive property of multiplication over addition (e.g., 7 × 8 = 7 × (5 + 3) = (7 × 5) + (7 × 3). At years 5–6, **represent** multiplication using the area model, and make **connections** with place value (e.g., 34 × 7 = 30 × 7 + 4 × 7). **Explain** and demonstrate: › the vertical-column method for division and multiplication, ensuring students understand and practise the procedure and **connect** with place value, known facts, and estimation › making estimates or mental calculations by connecting to place value, partitioning, and known facts. Have students **investigate**: › decoding and solving word problems, **representing** them as equations › multiplication and division in measurement and proportional reasoning situations › multiplication to count different combinations (e.g., "If I have 4 tops and 3 pairs of shorts, how many different outfits can I make?")
	› divide up to a three-digit whole number by a one-digit divisor, with no remainder (e.g., 65 ÷ 5)	› divide up to a three-digit whole number by a one-digit divisor, with a remainder (e.g., 83 ÷ 5 = 16, remainder 3)	› divide up to a four-digit whole number by a one-digit divisor, with a remainder (e.g., 198 ÷ 7, 4154 ÷ 8)	
			› use the order of operations rule with grouping, addition, subtraction, multiplication, and division	Use worked examples to demonstrate a step-by-step layout with one equal sign per line. Have students **investigate**: › decoding and solving word problems, deciding which operation to use and why › the distributive property of multiplication over addition and subtraction (e.g., 6 × 18 = 6 × (20 – 2) = (6 × 20) – (6 × 2). **Explain** the commutative, associative, and identity properties, and justify which operations they work for and which they don't.
Rational numbers	› identify, read, write, and represent tenths as fractions and decimals	› identify, read, write, and represent tenths and hundredths as fractions and decimals	› identify, read, write, and represent fractions, decimals (to two places), and related percentages	**Represent** and compare fractions, decimals, and percentages using continuous materials (double number lines, fraction walls, 100s squares). Have students practise saying, reading, and writing decimals using decimal PV houses. **Explain** and **represent** decimal tenths as a fraction with the denominator as 10, and percentages and decimals (to two places) as a fraction with the denominator of 100. **Investigate** situations where decimals are used (e.g., in measurements at a sports day).
	› compare and order tenths as fractions and decimals, and convert decimal tenths to fractions (e.g., 0.3 = $\frac{3}{10}$)	› compare and order tenths and hundredths as fractions and decimals, and convert decimal tenths and hundredths to fractions	› compare and order fractions, decimals (to two places), and percentages, and convert decimals and percentages to fractions	

	During year 4 *Informed by prior learning, teach students to:*	During year 5 *Informed by prior learning, teach students to:*	During year 6 *Informed by prior learning, teach students to:*	Teaching considerations
Rational numbers	› divide whole numbers by 10 to make decimals	› divide whole numbers by 10 and 100 to make decimals	› multiply and divide numbers by 10 and 100 to make decimals and whole numbers (e.g., 1.3 × 10 = 13)	Use decimal PV houses to **generalise** that multiplying by 10 moves each digit in a number one place to the left (increasing the place value of the digit), and dividing moves each digit one place to the right (decreasing the place value of the digit).
	› for fractions with related denominators of 2, 4, and 8, 3 and 6, or 5 and 10: – compare and order the fractions – identify when two fractions are equivalent by directly comparing them, noticing the simplest form (e.g., $\frac{3}{6} = \frac{1}{2}$, which is the simplest form)	› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100: – compare and order the fractions – identify when two fractions are equivalent	› for fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100: – compare and order the fractions – identify when two fractions are equivalent – represent the fractions in their simplest form	Use fraction walls (equivalence materials) to **represent** and **investigate** the relationship between the denominator and numerator in a fraction and how we can use this to simplify the fraction. Make **connections** with known facts such as halving and dividing by 4. Count forwards and backwards in fractions, and place fractions on marked and unmarked number lines.
	› convert (using number lines) between mixed numbers and improper fractions with denominators of 2, 3, 4, 5, 6, 8, and 10	› convert between mixed numbers and improper fractions with denominators of up to 10	› convert between mixed numbers and improper fractions	**Represent** improper fractions using words and materials, and place them on a number line. At years 5–6, **explain** conversion as division with a remainder (e.g., $\frac{11}{4} = 2\frac{3}{4}$ (11 divided by 4 = 2 r 3) or multiplication plus a remainder (e.g., $1\frac{1}{5} = \frac{6}{5}$ (1 × 5 + 1).
	› find a unit fraction of a whole number, using multiplication or division facts and where the answer is a whole number (e.g., $\frac{1}{5}$ of 40) › identify, from a unit fraction part of a set, the whole set	› find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., $\frac{2}{3}$ of 24) › identify, from a fractional part of a set, the whole set	› find a fraction or percentage of a whole number where the answer is a whole number (e.g., $\frac{3}{8}$ of 48; 30% of $150) › identify, from a fractional part of a set, the whole set	Use bar models, diagrams, or paper strips to **represent** equal parts of a whole. At year 4, **represent** parts of a whole set using discrete materials to make equal groups. At years 5–6, **connect** percentages and fractions of a whole to known facts and benchmarks (e.g., 25% and dividing by 4).
	› add and subtract fractions with the same denominators to make up to one whole (e.g., $\frac{3}{8} + \frac{3}{8} + \frac{2}{8} = \frac{8}{8} = 1$)	› add and subtract fractions with the same denominators, including to make more than one whole	› add and subtract fractions with the same or related denominators (e.g., $\frac{1}{4} + \frac{1}{8}$)	**Represent** the addition and subtraction of fractions using fraction walls, number lines, and equations. At years 4–5, **explain** that, when adding and subtracting fractions with the same denominator, the numerators are added or subtracted but the denominator stays the same. At year 6, **explain** how to use equivalent fractions to rename fractions so that they all have the same denominator. Then add or subtract the numerators.
	› add and subtract decimals to one decimal place (e.g., 1.3 + 0.2 = 1.5)	› add and subtract whole numbers and decimals to two decimal places (e.g., 32.55 – 21.21 = 11.34)	› add and subtract whole numbers and decimals to two decimal places (e.g., 250.11 + 135.29 = 385.4)	**Explain** and demonstrate both the horizontal method for **representing** an equation and the vertical-column method for addition or subtraction. **Investigate** and **connect** the addition and subtraction of decimals in measurement situations. At year 4, use number lines and decimals to add and subtract tenths, **connecting** tenths as fractions with tenths as decimals. At years 5–6, **connect** to methods of adding and subtracting whole numbers.
	› use doubling or halving to scale a quantity (e.g., to double or halve a recipe)	› use known multiplication facts to scale a quantity	› use known multiplication and division facts to scale a quantity	**Represent** multiplicative relationships using diagrams, materials, and bar models. Use problems such as "If this recipe feeds 4 people, how much of each ingredient do we need to feed 20 people"?

	During year 4 *Informed by prior learning, teach students to:*	During year 5 *Informed by prior learning, teach students to:*	During year 6 *Informed by prior learning, teach students to:*
Financial mathematics	› make amounts of money using dollars and cents (e.g., to make 3 dollars and 70 cents)	› represent money values in multiple ways using notes and coins	› solve problems involving purchases (e.g., ensuring they have enough money) › create simple financial plans (e.g., shopping lists, a family budget)
	› estimate and calculate the total cost and change for items costing whole-dollar amounts.	› estimate to the nearest dollar and calculate the total cost of items costing dollars and cents, and the change from the nearest ten dollars.	› calculate 10%, 25%, and 50% of whole-dollar amounts (e.g., 50% of $280).

Teaching considerations
Have students practise grouping denominations and making amounts using play money, **connecting** with place value, skip counting, and multiplication. **Investigate** authentic financial situations and **represent** findings using equations, spreadsheets, and tables.
Investigate practical situations involving calculating costs and giving change. At year 6: › use bar models to **represent** percentages of whole-dollar amounts, and connect to equivalent fractions › **explain** the procedure of dividing a whole by 10 to find 10%, 2 to find 50%, or 4 to find 25%.

Algebra

	During year 4 *Informed by prior learning, teach students to:*	**During year 5** *Informed by prior learning, teach students to:*	**During year 6** *Informed by prior learning, teach students to:*
Equations and relationships	› form and solve true or false number sentences and open number sentences involving multiplication and division, using an understanding of the equal sign (e.g., 5 × __ = 20; __ ÷ 3 = 6)	› form and solve true or false number sentences and open number sentences involving all four operations (e.g., 674 + 56 – __ = 671)	› form and solve true or false number sentences and open number sentences involving all four operations, using an understanding of equality or inequality (e.g., 8 × 7 < 8 × 5 + 8 (T or F?)
	› recognise and describe the rule for a growing pattern using words, tables, and diagrams, and make conjectures about further elements in the pattern	› use tables to recognise the relationship between the ordinal position and its corresponding element in a growing pattern, develop a rule for the pattern in words, and make conjectures about further elements or terms in the pattern	› use tables, XY graphs, and diagrams to recognise relationships in a linear pattern, develop a rule for the pattern in words (i.e., that there is a constant amount of change between consecutive elements or terms), and make conjectures about further elements in the pattern
Algorithmic thinking	› create and use an algorithm for generating a pattern or pathway.	› create and use an algorithm for generating a pattern, procedure, or pathway.	› create and use algorithms for making decisions that involve clear choices.

Teaching considerations
Represent the equal sign as 'the same as' to demonstrate it is a symbol of equivalence. **Explain** the difference between an expression (e.g., 4 × 5), an equation (e.g., 4 × 5 = 20), and an inequality (e.g., 4 × 5 < 4 × 6). Have students practise the use of equal and inequality symbols. **Investigate** inverse operations to find missing numbers in equations.
Explain vocabulary in relation to patterns (e.g., ordinal, element, term, position, rule) and how to record the position and term for each element in a pattern. **Investigate** visual patterns (e.g., tivaevae), making block patterns and **representing** patterns using pictures and materials.
Represent a procedure as a sequence of step-by-step instructions (an algorithm). Follow the sequence by 'acting it out', asking students to describe and record each step. **Investigate** giving directions for, or describing, the most efficient pathway on a maze or map, and sorting numbers according to a set of instructions (e.g., "Sort the odd numbers … the multiples of 5"). **Explain** and **justify** how a procedure has been broken into steps, the order of the steps, whether there were any errors or omissions, and, if so, how they were corrected. At years 4–5, **investigate** creating a sequence of instructions (e.g., to draw a polygon or move through a maze), using digital tools or on paper. **Connect** with geometry when giving directions or describing pathways. At year 5, **connect** algorithmic thinking to a procedure for an operation (e.g., for multiplying two numbers). At years 5–6, **investigate** identifying the transformations used to create geometric patterns. At year 6, **investigate** using classification diagrams to identify an object, a shape, or data based on multiple characteristics.

Measurement

	During year 4 *Informed by prior learning, teach students to:*	During year 5 *Informed by prior learning, teach students to:*	During year 6 *Informed by prior learning, teach students to:*	Teaching considerations
Measuring	› measure body parts (e.g., the arm) or familiar objects and use these as benchmarks to estimate and then measure length, mass (weight), capacity, and duration, using appropriate metric or time-based units	› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric or time-based units or a combination of units	› estimate and then accurately measure length, mass (weight), capacity, temperature, and duration, using appropriate metric or time-based units or a combination of units	**Investigate** practical measuring situations, and have students practise the accurate use and reading of rulers, scales, timers, thermometers, and measuring jugs. **Explain** and accurately measure: › at year 4, centimetres, metres, grams, kilograms, and litres, connecting with half units (e.g., 500 mL = 0.5 L) › at years 5–6, centimetres, metres, millimetres, grams, kilograms, litres, millilitres, and degrees Celsius. **Connect** reading a measuring tool with rounding to the nearest given unit (e.g., 3.6 cm to the nearest cm). Discuss the meaning of measurements in context. Explain benchmarks and prompt students to develop them (e.g., "A big step is about a metre, so roughly how long is our classroom?")
	› use appropriate units to describe length, mass (weight), capacity, and time	› use the appropriate tool for a measurement and the appropriate unit for the attribute being measured	› select and use the appropriate tool for a measurement and the appropriate unit for the attribute being measured	**Explain** and **justify** the use of appropriate metric units or tools for measuring a given attribute with the precision necessary for the problem, noting that using smaller units provides more accuracy.
	› use the metric measurement system to explore relationships between units	› use the metric measurement system to explore relationships between units, including relationships represented by benchmark fractions and decimals	› convert between common metric units for length, mass (weight), and capacity, and use decimals to express parts of wholes in measurements	**Explain** measurement prefixes (e.g., milli-, centi-), how they **connect** metric units, and how they are based on powers of ten and relate to place value. **Investigate** how measures can be partitioned and combined like other numbers, and how smaller units are created by equally partitioning larger units.
	› recognise that angles can be measured in degrees, using 90, 180, and 360 degrees as benchmarks	› describe angles using the terms acute, right, obtuse, straight, and reflex, comparing them with benchmarks of 90, 180, and 360 degrees	› visualise, measure, and draw (to the nearest degree) the amount of turn in angles up to 360 degrees	**Investigate** different angles using physical and digital tools and angles in the environment, and comparing and classifying them as acute, right, reflex, or obtuse. Make **connections** between angles, fractions of a circle, and turns. At year 6, **explain**, demonstrate, and have students practise estimating angles and measuring and drawing them using a protractor.
	› tell the time to the nearest 5 minutes, using the language of 'minutes past the hour' and 'to the hour'	› describe the differences in duration between units of time (e.g., days and weeks, months, and years), and solve duration-of-time problems involving 'am' and 'pm' notation	› convert between units of time and solve duration-of-time problems, in both 12- and 24-hour time systems	**Represent** time using: › digital and analogue clocks (at year 4), to practise telling the time › analogue and digital forms (e.g., "It's 12:45, or a quarter to one.") **Investigate** using calendars, timetables, schedules, and number lines to work out the time between two events or the duration of an event. Explore solar calendars (e.g., Roman, Gregorian) and lunar calendars (e.g., maramataka Māori, Chinese). **Explain** subtracting for duration and inclusive counting (e.g., "For the number of days between now and next Tuesday, start counting from today"). **Explain** relationships between the units of time (e.g., 60 seconds to the minute, 60 minutes to the hour, 24 hours in a day, 365 days in a year), and use them to convert between units of time.

	During year 4 *Informed by prior learning, teach students to:*	During year 5 *Informed by prior learning, teach students to:*	During year 6 *Informed by prior learning, teach students to:*	Teaching considerations
Perimeter, area, and volume	› visualise, estimate, and measure: – the perimeter of polygons, using metric units (cm and m) – the area of shapes covered with squares or half squares – the volume of shapes filled with centicubes, taking note of layers and stacking.	› visualise, estimate, and calculate: – the perimeter of regular polygons (in m, cm, and mm) – the area of shapes covered with squares or partial squares – the volume of rectangular prisms filled with centicubes, taking note of layers and stacking.	› visualise, estimate, and calculate the area of rectangles and right-angled triangles (in cm^2 and m^2) and the volume of rectangular prisms (in cm^3), by applying multiplication.	**Investigate** practical measuring situations and **connect**: › finding area with multiplication arrays › finding area and volume with the commutative property of multiplication › how part-units can be combined using number concepts, when finding the area of a shape › the area of a right-angled triangle with half the area of a square. Have students **represent** written methods for calculating, with clearly laid-out working.

Geometry

	During year 4 *Informed by prior learning, teach students to:*	During year 5 *Informed by prior learning, teach students to:*	During year 6 *Informed by prior learning, teach students to:*	Teaching considerations
Shapes	› identify, classify, and describe the attributes of polygons (including triangles and quadrilaterals) using properties of shapes, including line and rotational symmetry	› identify, classify, and describe the attributes of: – regular and irregular polygons, using edges, vertices, and angles – prisms, using cross sections, faces, edges, and vertices	› identify, classify, and explain similarities and differences between: – 2D shapes, including different types of triangle – prisms and pyramids	Use a range of 2D and 3D shapes, including tactile shapes, diagrams, student-made constructions, and digital shapes. **Investigate** line and rotational symmetry using mirrors and tracing paper. **Connect** to algorithmic thinking by making classification diagrams for classifying shapes.
	› compare angles in 2D shapes, classifying them as equal to, smaller than, or larger than a right angle	› identify and describe parallel and perpendicular lines, including those forming the sides of polygons	› identify and describe the interior angles of triangles and quadrilaterals	**Investigate** interior angles using digital tools and paper shapes to **generalise** that the interior angles of a triangle add to 180° and those of a quadrilateral add to 360°. **Connect** these understandings to ideas about right angles, straight lines, and full turns.
Spatial reasoning	› identify the 2D shapes that compose 3D shapes (e.g., a triangular prism is made from two triangles and three rectangles)	› visualise 3D shapes and connect them with nets, 2D diagrams, verbal descriptions, and the same shapes drawn from different perspectives	› visualise and draw nets for rectangular prisms	**Represent** 3D shapes using digital tools, sketches, blocks, and student-made constructions. **Investigate** nets that will or will not fold, and match solid shapes with nets.
	› visualise, predict, and identify which shape is a reflection, rotation, or translation of a given 2D shape	› resize (enlarge or reduce) a 2D shape	› visualise, create, and describe 2D geometric patterns and tessellations, using rotation, reflection, and translation and identifying the properties of shapes that do not change	**Investigate** using 2D shapes, squared paper, mirrors, and tracing paper to make and test conjectures about the effects of transformations. At year 5, use a grid to scale a shape and connect the scaling with multiplication or division. At year 6, **generalise** the properties of shapes that do not change when transformed (e.g., "Which properties of a square stay the same when we rotate it 90 degrees?")

	During year 4 *Informed by prior learning, teach students to:*	**During year 5** *Informed by prior learning, teach students to:*	**During year 6** *Informed by prior learning, teach students to:*
Pathways	› use grid references to identify regions and plot positions on a grid map › interpret and describe pathways, including those involving half and quarter turns and the distance travelled.	› interpret and create grid maps to plot positions and pathways, using grid references and directional language, including the four main compass points.	› interpret and create grid references and simple scales on maps › use directional language, including the four main compass points, turn (in degrees), and distance (in m, km) to locate and describe positions and pathways.

Statistics

	During year 4 *Informed by prior learning, teach students to:*	**During year 5** *Informed by prior learning, teach students to:*	**During year 6** *Informed by prior learning, teach students to:*
Problem	› use multivariate data to investigate summary and comparison situations with categorical and discrete numerical data, by: – posing an investigative question that can be answered with data – making conjectures or assertions about expected findings		› use multivariate data to investigate summary, comparison, and time-series situations, by: – posing an investigative question that can be answered with data – making conjectures or assertions about expected findings
Plan	› plan how to collect primary data to support answering the investigative question, including: – deciding on the group of interest – deciding on the variable or variables for which data will be collected – taking account of ethical practices in data collection		› plan how to collect primary data or how to use provided data, including identifying the variables of interest and, for provided data: – identifying who the data was collected from – identifying the original investigator's purpose for collecting the data – deciding if the source is reliable (e.g., by checking if survey questions appear to be biased towards a particular point of view)

Teaching considerations
Investigate different types of maps (e.g., schematic, topographical, and digital maps). **Explain** pathways using directional language, including te reo Māori (e.g., whakamua/forwards, whakamuri/backwards, whakamauī/to the left, whakamatau/to the right, raki/north, tonga/south, rāwhiti/east, uru/west). **Connect** compass points and directional language with turns and angles, and simple scales with proportional reasoning.

Teaching considerations
Show, with student input, how to: › pose summary and comparison **investigative** questions › pose time-series **investigative** questions (at year 6). **Connect** questions to areas of interest and value to the students and their communities.
Show, with student input, how to: › ask interrogative questions about sources and ethical practices › develop and closely examine survey or data-collection questions › define or establish measures for variables › identify 'who, what, where, when, and how' when using secondary datasets.

	During year 4 ***Informed by prior learning, teach students to:***	**During year 5** ***Informed by prior learning, teach students to:***	**During year 6** ***Informed by prior learning, teach students to:***
Data	› use a variety of tools to collect the data, and check for errors in it	› use a variety of tools to collect the data, check for errors in it, and correct them by re-collecting the data, if possible	› collect primary data and check for errors, and provide information about variables in secondary data (e.g., how data was collected for them and possible outcomes for them)
Analysis	› create and describe data visualisations to make meaning from the data, with statements including the name of the variable	› create and describe data visualisations to make meaning from the data, with statements including the names of the variable and group of interest	› create and describe a variety of data visualisations to make meaning from the data, identifying features, patterns, and trends in context, and including the variable and group of interest
Conclusion	› choose descriptive statements that best answer the investigative question, reflecting on findings and how they compare with initial conjectures or assertions	› answer the investigative question, comparing findings with initial conjectures or assertions and their existing knowledge of the world	
Statistical literacy	› check the statements that others make about data to see if they make sense, using information to clarify or correct statements where needed.	› check and, if necessary, improve the statements others make about data, including data from two or more sources.	› identify, explain, check, and, if necessary, improve features in others' data investigations (e.g., biased survey questions, misleading information or statements).

Teaching considerations
Show, with student input, how to: › use a range of **representations** for recording data › identify what errors in data look like. **Connect** multiple variables for individuals, explaining that most datasets use a table design in which each row focuses on an individual and each column includes the data on multiple individuals for one variable.
Show, with student input, how to: › **represent** and analyse data visualisations, creating them at first by hand and then with digital tools › identify the different features of data that the data visualisation reveals and how to describe them › read the data, and read 'between' the data. **Explain** how different data visualisations have different features and how to describe them in context (e.g., in relation to frequency, modes, modal groups, patterns, trends, values for numerical variables).
Show, with student input, how to: › choose the best descriptive statements that answer an investigative question › prepare their findings and explain them to others.
Show, with student input, how to: › identify misleading data visualisations, match others' data visualisations with their statements, and check the claims made by others › interpret pie graphs (but not how to create them) › **explain** and **justify** the effectiveness of data visualisations in representing others' findings, using interrogative questions.

Probability

<table>
<tr><th></th><th>During year 4
Informed by prior learning, teach students to:</th><th>During year 5
Informed by prior learning, teach students to:</th><th>During year 6
Informed by prior learning, teach students to:</th></tr>
<tr><td>Probability investigations</td><td>› engage in chance-based investigations with equally likely outcomes by:
– posing an investigative question
– anticipating and then identifying possible outcomes for the investigative question
– generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome
– creating data visualisations for possible outcomes
– describing what these visualisations show
– finding probabilities as fractions
– answering the investigative question
– reflecting on anticipated outcomes</td><td colspan="2">› engage in chance-based investigations, including those with not equally likely outcomes, by:
– posing an investigative question
– anticipating and then identifying possible outcomes for the investigative question
– generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome
– creating data visualisations for possible outcomes
– describing what these visualisations show
– finding probabilities as fractions
– answering the investigative question
– reflecting on anticipated outcomes
– (at year 6) comparing findings from the probability experiment and associated theoretical probabilities, if the theoretical model exists</td></tr>
<tr><td>Critical thinking in probability</td><td>› agree or disagree with others' conclusions about chance-based investigations.</td><td>› evaluate others' statements about chance-based investigations, with justification.</td><td>› identify, explain, and check others' statements about chance-based investigations, referring to evidence.</td></tr>
</table>

Teaching considerations
Investigate everyday chance-based situations in order to explore and experience the chance, randomness, variation, and distribution of outcomes. Use digital tools to conduct a large number of trials in order to see what a probability estimate and probability distributions look like. Support students to **represent**: › probability outcomes (theoretical and experimental) using lists, tables, tally charts, visualisations of distributions, words, and numbers › the chance of an outcome occurring using fractions, decimals, and percentages. **Connect** investigative questions to outcomes and with all possible ways to get the outcomes. **Connect** anticipated outcomes with theoretical and experimental distributions.
Show, with student input, how to: › match the results of chance-based investigations with statements, and check the claims in others' investigations › **explain** and **justify** the statements made by others about chance-based investigations, using interrogative questions.

The language of mathematics and statistics: Phase 2

	Year 4 *Students will know the following new words:*	**Year 5** *Students will know the following new words:*	**Year 6** *Students will know the following new words:*
Number	› addend › convert › decimal › decimal place › decimal point › improper fraction › mixed number › rename › scale › tenth	› change › divisor, dividend, quotient, remainder › factor › hundredth › multiple › product › proportion	› efficient › inverse operation › percentage › simplest form › square number › thousandth
Algebra	› conjecture › relationship	› algorithm › corresponding element › procedure	› constant › equality, inequality › linear pattern › XY graph
Measurement	› angle › benchmark › degree › kilogram › minutes past, minutes to	› a.m., p.m. › acute angle › attribute › degrees celsius › kilometre, millimetre › obtuse, reflex, right, or straight angle › timetable	› cubic centimetre, cubic metre › protractor › square centimetre, square metre

	Year 4 *Students will know the following new words:*	**Year 5** *Students will know the following new words:*	**Year 6** *Students will know the following new words:*
Geometry	› grid reference › rotational symmetry	› compass points › cross section › net › parallel or perpendicular line › perspective › prism › regular or irregular polygon › resize, enlarge, reduce	› centre of rotation › clockwise, anticlockwise › interior angle › map scale › right-angled, equilateral, isosceles, or scalene triangle › tessellation
Statistics	› analysis › assertion › investigative question › conclusion	› categorical › data visualisation › discrete numerical › group of interest › source	› comparison or summary investigative question › feature › misleading › mode › primary or secondary data › trend
Probability	› chance-based investigation › equally likely outcome › probability experiment	› evaluate › not an equally likely outcome	› evidence

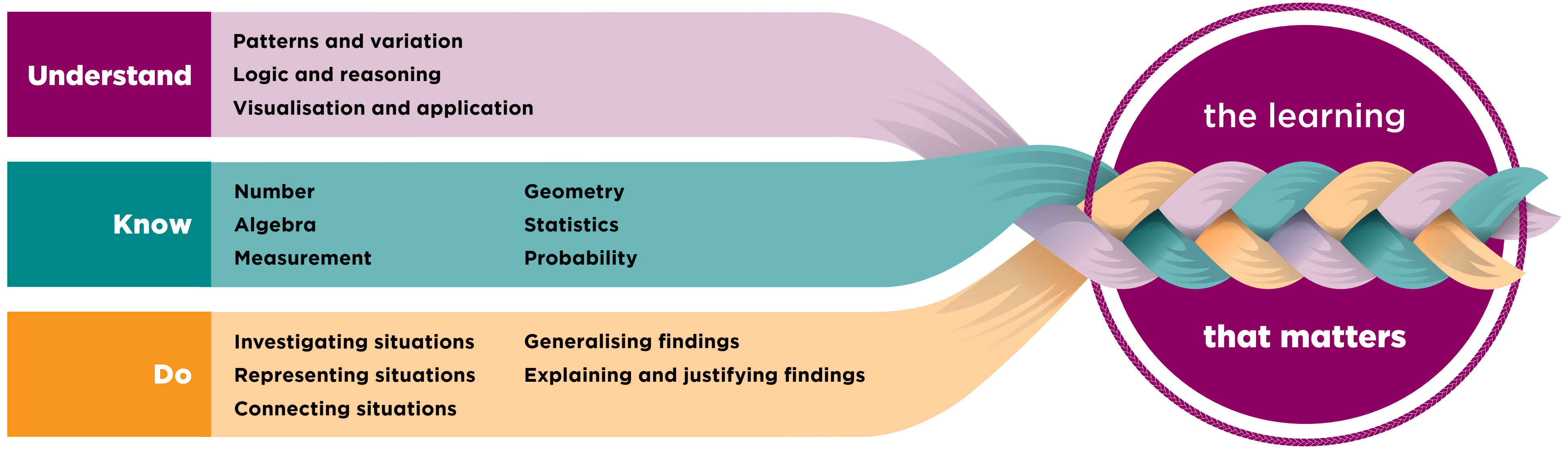
Phase 3 Years 7–8

Progress outcome by the end of year 8

Seeing ourselves in the wider world and advocating with and for others
Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu

The critical focus of phase 3 is for all students to see themselves in the wider world and to advocate with and for others. Students use logic and reasoning to identify and solve problems, make connections between mathematical and statistical concepts, and investigate patterns and variation. They communicate mathematically and statistically, using notation, conventions, and vocabulary to clearly explain and justify their approaches to solving problems. Students select, use, and adapt representations to visualise and extend their reasoning (e.g., number lines to represent integers, equations to represent linear patterns). They make generalisations and identify unknown quantities (e.g., the size of angles) and use data visualisations to investigate claims and make conjectures.

The phase 3 progress outcome describes the understanding, knowledge, and processes that students have multiple opportunities to develop over the phase.



The phase 2 progress outcome is found on the following two pages.

Understand

As students build knowledge through their use of the mathematical and statistical processes, they expand their understanding of the following.

Patterns and variation | Ngā ia auau me ngā rerekētanga

The world is full of patterns and is defined by a multitude of relationships in which change and variation occur. Mathematics and statistics provide structures that are useful for noticing, exploring, and describing different types of patterns and relationships, enabling us to generate insights or make conjectures.

Logic and reasoning | Te whakaaro arorau me te whakaaroaro

By engaging with mathematical concepts, we develop logical reasoning and critical thinking skills that enable us to evaluate information, question assumptions, and present arguments with clarity. Statistical reasoning from observation and theory allows us to differentiate what is probable from what is possible and to draw reliable conclusions about what is reasonable.

Visualisation and application | Te whakakite me te whakatinana

The visualisation of mathematical and statistical ideas profoundly influences how we perceive, understand, and interact with abstract concepts. Application in mathematics and statistics involves creating structures and processes that help us understand complex situations, enabling better decision making and communication of ideas.

Know

Number | Mātauranga tau

By the end of this phase, students know that some numbers have special properties (e.g., primes, composites, squares, square roots, cubes). A fraction can describe a proportional relationship between two amounts. Every fraction can be represented by an infinite set of equivalent fractions that occupy the same point on the number line. Multiplying a fraction by an equivalent form of 1, such as $\frac{3}{3}$, results in an equivalent fraction that can be useful for comparing, adding, and subtracting fractions. Decimals continue the place-value system using negative powers of ten. They can be terminating, repeating and infinite, or non-repeating and infinite.

Students come to know that integers are positive and negative whole numbers and include zero. To compare relative magnitude, integers, fractions, and decimals can be represented on a number line. There are real-life situations described by quantities less than zero (e.g., temperature, below sea level, debt), and these quantities can be operated on.

Students also come to know that when evaluating or forming expressions, the order of operations is important. Operations inside brackets (i.e., grouped together) are done first, then powers or exponents. If there are multiplication and division, these are then done in left-to-right order; finally, addition and subtraction are also done in left-to-right order. Division can result in a remainder expressed as a whole number, fraction, or decimal.

Algebra | Taurangi

By the end of this phase, students know that the inverse property applies to addition and multiplication. Inequalities can also include 'or equal to' (≤, ≥) to show a relationship that allows for the possibility of equality. In algebra, a variable can be used to represent an unknown number, a quantity that can vary or change (e.g., y= 3x + 4; A = bh), or a specific unknown value to be solved for (e.g., 3a = 18). In algebra, there are conventional ways of writing multiplication and division.

Students also come to know that linear patterns have a constant increase or decrease and their XY graphs are straight lines. Not all patterns are linear. Algorithms help solve problems in a systematic way. Their instructions are created, tested, and revised.

Measurement | Ine

By the end of this phase, students know that in the metric system there are base measurements, with prefixes added to show the size of units. A measurement can be converted from smaller to bigger units, and vice versa, by multiplying or dividing by powers of 10. Length is a one-dimensional measure, area is a two-dimensional measure, and volume is a three-dimensional measure. This is apparent in the notation of units (e.g., cm, cm^2, cm^3). Shapes can be decomposed or recomposed to help us find their measurements (e.g., their perimeters, areas, volumes).

Geometry | Āhuahanga

By the end of this phase, students know that the spatial properties of simple polygons and polyhedra can also apply to more complex two- and three-dimensional shapes. Properties of two- and three-dimensional shapes that do not change under a transformation are called invariant. Unknown angles can be found using the properties of angles on a straight line, angles at a point, vertically opposite angles, and interior angles in triangles and quadrilaterals. Viewing objects from different angles gives different perspectives, which can be represented in models and diagrams. Position, direction, and pathways can be described using scale, compass points, and environmental features. Coordinate systems and maps can express position, direction, and pathways.

Statistics | Tauanga

By the end of this phase, students know that data collection and use involves a responsibility to protect the rights of people (in relation to data about them) and the ethical use and interpretation of data. People need to understand who they are giving data to and why, before they agree to contribute to a dataset. The statistical enquiry cycle (PPDAC) can be used to conduct data-based investigations about the wider community. There are different types of questions used when undertaking statistical investigations: investigative (summary, comparison, relationship, or time-series), survey, data-collection, interrogative, and analysis questions. Data visualisations show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information. A distribution is formed from all the possible values of a variable and their frequencies. A relationship investigation looks for a connection between paired numerical or paired categorical variables. Conjectures or assertions may not be reflected in the data, and so may need to be revised or abandoned.

Probability | Tūponotanga

By the end of this phase, students know that a probability experiment involves repeated trials. Results from sets of repeated trials for the same experiment may vary. Some chance-based situations, such as rolling a weighted dice, can only be explored by probability experiments. Estimates of probabilities from experiments should be based on a very large number of trials (the 'law of large numbers'). The estimated probability of an event from an experiment equals the relative frequency for that event.

Students come to know that if all possible outcomes in a chance-based situation are assumed to be equally likely, the probability of an event equals the number of ways the event can happen divided by the total number of possible outcomes. The statistical enquiry cycle (PPDAC) can be used to conduct probability experiments. For a given situation, probability estimates from experiments and outcomes for theoretical probability models will differ. Probability distributions from experiments and theoretical models will also differ.

Do

Investigating situations | Te tūhura pūāhua

By the end of this phase, students can pose a question for investigation, find entry points for addressing the question, and plan an investigation pathway and follow it in an organised way. They can identify relevant prior knowledge, conditions, assumptions, and relationships. They can monitor and evaluate progress, adjusting the investigation pathway if necessary, and make sense of outcomes or conclusions in light of a given situation and context.

Representing situations | Te whakaata pūāhua

By the end of this phase, students can use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends. They use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments. They select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations. They use visualisation to mentally represent and manipulate relationships, objects, and ideas.

Connecting situations | Te tūhono pūāhua

By the end of this phase, students can suggest connections between concepts, ideas, approaches, and representations. They connect new ideas to things they already know. They make connections to ideas in other learning areas and with diverse contexts.

Generalising findings | Te whakatauwhānui i ngā kitenga

By the end of this phase, students can notice and explore patterns, structure, and regularity and make conjectures about them. They identify relationships, including similarities, differences, and new connections. They represent specific instances and look for when conjectures about them might be applied in another situation or always be true. They test conjectures, using reasoning and counterexamples to decide if they are true or not. They use appropriate symbols to express generalisations.

Explaining and justifying findings | Te whakamārama me te parahau i ngā kitenga

By the end of this phase, students can make statements and give explanations inductively based on observations or data. They make deductions based on knowledge, definitions, and rules. They critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand. They use evidence, reasoning, and proofs to explain why they agree or disagree with statements. They develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others. They present reasoned, coherent explanations and arguments for an idea, solution, or process.

Phase 3 Years 7–8

Teaching sequence

Seeing ourselves in the wider world and advocating with and for others
Te aro atu ki te ao whānui me te kōkiri kaupapa hei hāpai tahi i ētahi atu

This section describes how the components of a comprehensive teaching and learning programme for the mathematics and statistics learning area are used during the third phase of learning at school.

Throughout phase 3, demonstrate, highlight, and affirm an attitude of exploration, enthusiasm, and curiosity towards mathematical and statistical endeavour and challenge, holding high expectations for every student. In this phase, students critically reflect on others' reasoning, evaluating their logic and asking questions for clarification. To promote this, facilitate ongoing discussions and reflections about established expectations for interactions in mathematical and statistical learning, reinforcing that all students will be involved. Support increasing agency for students in making decisions about investigations and problem solving (e.g., while planning their approach, selecting representations, justifying their findings).

Continuously monitor students' cognitive load, reasoning, questions, and use of representations, and respond quickly to address any issues and misconceptions. Ensure teaching builds on what students already understand, know, and can do.

Explicit teaching

- Use worked examples and break down new learning into clearly explained, manageable steps. Show students efficient written and mental methods. Use examples where there may be an error, misconception, or missing step, to support students to develop critical-analysis and reasoning skills.
- Plan for students to actively recall learning, practise new procedures and processes, and make connections with prior learning. Provide regular opportunities to practise, so that students maintain their automatic recall of facts and continue to develop procedural fluency and reasoning. Following sufficient blocked practice to achieve proficiency, provide practice opportunities that interleave a mixture of operations or approaches, rather than working on only one concept or procedure in a specific way.

Positive relationships with mathematics and statistics

- Provide authentic tasks that reflect students' experiences, interests, and the wider world.
- Demonstrate and teach strategies for perseverance (e.g., trying another way, drawing a diagram, talking about the task with another student).

Rich tasks

- Design investigations where students experience rich mathematical situations, as well as investigations where students use their findings to make decisions in their lives (e.g., making a savings plan). When planning an investigation, help students to identify appropriate questions, as well as the mathematical and statistical concepts, procedures, and representations they will need.
- Design tasks that have multiple entry and exit points and more than one solution or pathway.
- Teach problem-solving and investigation strategies such as:
  - making sense of the problem by drawing a diagram or considering previously solved problems to identify strategies that can be reapplied
  - trying some sequential numbers, recording the results in a table, and looking for patterns
  - identifying key information in the problem and connecting it to prior knowledge
  - translating a word problem into a linear equation, to solve for an unknown quantity
  - recording calculations in an organised way, using correct mathematical notation
  - checking the reasonableness of findings.

Communication in mathematics and statistics

- Set up opportunities for students to actively listen, reflect, and build on each other's thinking and learning. Use discourse-based tools and a range of open questions to facilitate productive and thought-provoking discussions. Over the phase, encourage students to convert their observations into a conjecture or claim and to use evidence to justify their claims and findings. Plan to balance 'teacher talk' with opportunities for rich, extended student interactions and discussions.
- Encourage students to select and use representations that best support the learning purpose, including graphs, tables, and equations. Over the phase, support them to increasingly use equations to represent their reasoning and to visualise situations by drawing a diagram, which can give them a way into a problem.
- Teach and use mathematical and statistical vocabulary and concepts. Ensure students connect the correct vocabulary to the learning purpose and problem (e.g., by using the Frayer model's four quadrants: definition, characteristics, example, non-example). Where possible, draw on students' first and heritage languages so that they can use their languages as a resource to connect their thinking and learning.
- Prompt students to share their thinking when using visualisation to represent and manipulate relationships, shapes, quantities, and data (e.g., to predict or deduce the effect of a transformation; view a solid shape from different perspectives; use coordinate pairs and locations; identify terms in a growing pattern).

Number

	During year 7 *Informed by prior learning, teach students to:*	**During year 8** *Informed by prior learning, teach students to:*	**Teaching considerations**
Number structure	› identify, read, write, compare, and order whole numbers using powers of 10 (e.g., 10,000 = 10^4)	› identify, read, write, compare, and order whole numbers and decimals using powers of 10 (e.g., 0.01 = $\frac{1}{100}$ = 10^{-2})	**Represent** and order numbers using place-value (PV) expanders or charts and number lines.
	› find the highest common factor (HCF) of two numbers under 100, and find the least common multiple (LCM) of two numbers under 10	› use prime factorisation to represent a number and to find the HCF of two numbers	**Represent** factors using factor trees, or systematic lists. **Connect** HCFs to simplifying fractions, and LCMs to renaming fractions. **Generalise** conjectures about prime or composite numbers by investigating factors.
	› use exponents to represent repeated multiplication, and identify square roots of square numbers up to at least 100	› identify and describe the properties of prime and composite numbers up to at least 100 and cube numbers up to at least 125	**Investigate** and **generalise** divisibility tests for composite and prime numbers, and **connect** the results to square and cube numbers and square roots. **Investigate** and **explain** patterns in repeated multiplication and **represent** them using exponent notation. **Connect** prime and composite numbers with factors, and **represent** a number as a product of its prime factors (prime factorisation).
Operations	› use rounding and estimation to predict results and to check the reasonableness of calculations	› use rounding, estimation, and benchmarks to predict results and to check the reasonableness of calculations	**Explain** efficient methods for supporting estimation (e.g., when adding a long list of numbers, look for numbers that can be grouped and summed to roughly 10, 100, 1000). **Connect** operations to benchmarks to make estimates (e.g., 73% is roughly $\frac{3}{4}$). **Explain** and **justify** findings, by connecting to estimates and other checking methods such as using the inverse operation.
	› round whole numbers to any specified power of 10, and round decimals to the nearest tenth, hundredth, or whole number	› round whole numbers to any specified power of 10, and round decimals to the nearest tenth, hundredth, thousandth, or whole number	
	› recall multiplication facts to at least 10 × 10 and identify and describe the divisibility rules for 2, 3, 5, 9, and 10	› identify and describe the divisibility rules for 2–11	**Investigate** patterns in multiples in 100s boards and multiplication charts to **generalise** divisibility rules.

	During year 7 *Informed by prior learning, teach students to:*	**During year 8** *Informed by prior learning, teach students to:*	**Teaching considerations**
Operations	› multiply whole numbers		**Explain** and demonstrate efficient methods using worked examples, including: › the vertical-column method for division and multiplication, ensuring students understand and practise the procedure and **connect** with place value, known facts, and estimation › making estimates or mental calculations by connecting to place value, partitioning, and known facts. **Investigate**, **explain**, and **justify** which method (including the use of digital tools) best suits a given situation. Have students practise decoding and solving word problems and representing them as equations. Represent and make sense of remainders as fractions, as decimals, and when rounded to the nearest whole number.
	› divide whole numbers by one- or two-digit divisors (e.g., 327 ÷ 5 = 65.4 or 65 $\frac{2}{5}$)	› divide whole numbers (e.g., 327 ÷ 15 = 21.8 or 21 $\frac{4}{5}$)	
	› use the order of operations	› use the order of operations	Use worked examples to demonstrate a step-by-step layout with only one equal sign per line. Demonstrate how to use the mnemonic GEMA in relation to the order of operations: grouped, exponents, multiplicative (÷ and ×), additive (+ and –).
	› order, compare, and locate integers on a number line, and explore adding and subtracting integers	› order, compare, add, and subtract integers	**Generalise** that a positive number has an opposite negative number, and that when they are added, the answer is zero (e.g., 4 + –4 = 0). **Explain** how to: › find the number of steps between two given numbers on a number line (e.g., –5 and 7) › 'read' equations with integers on a number line (e.g., "To solve –9 + 8, start at –9 and move eight numbers in the positive direction.") › use inequality symbols to compare two integers (e.g., –5 < –3). **Investigate** adding and subtracting integers, using number lines and two-sided counters. *(continued on the next page)*

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Operations			*(continued from the previous page)* **Explain** the direction of movement on a number line when adding and subtracting integers, and **generalise** that: › adding a negative number makes the original number smaller (e.g., 4 + –3 = 1) › subtracting a negative number makes the original number larger (e.g., –7 – (–3) = –4). **Investigate** situations where negative integers are used (e.g., temperature, altitude, debt, profit and loss).
Rational numbers	› identify, read, write, and represent fractions, decimals (to three places), and percentages	› identify, read, write, and represent fractions, decimals, and percentages	**Explain** and **represent**: › percentages using 100s squares, › comparing or ordering fractions, decimals, and percentages using double number lines › decimals or percentages as fractions with denominators of tenths or hundredths, and then renamed to their simplest form › fractions in equivalent forms to support comparing, ordering, and converting. **Explain** and demonstrate converting a fraction to a decimal or percentage by connecting to the understanding of fractions as quotients (e.g., $\frac{5}{12}$ = 5 ÷ 12). **Connect** to known benchmarks when comparing and converting (e.g., $\frac{7}{12}$ is a little more than $\frac{6}{12}$, which is a half or 50%).
	› compare, order, and convert between fractions, decimals (to three places), and percentages	› compare, order, and convert between fractions, decimals, and percentages	
	› multiply and divide numbers by 10, 100, and 1,000	› multiply and divide numbers by powers of 10	**Represent** decimals using PV expanders or charts, and **generalise** that multiplying by a power of 10 moves each digit that number of places to the left, and dividing by a power of 10 moves each digit that number of places to the right.
	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	› find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	**Explain** simplifying fractions and finding equivalent fractions by using HCFs and LCMs.

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Rational numbers	› multiply fractions and decimals by whole numbers	› multiply fractions and decimals by whole numbers	**Explain** the vertical column method for multiplying decimals, making an estimate before calculating. **Connect** to the multiplicative identity to **generalise** that multiplying a whole number by a decimal less than one results in a product less than the original whole number.
	› find a percentage of a whole number, and find a whole amount, given a simple fraction or percentage (e.g., "25% is $100, what is the total amount?")	› find a percentage of a whole number, and find a whole amount, given a simple fraction or percentage (e.g., "75% is $45, what is the total amount?")	**Represent** situations involving percentages using bar models to show parts of a whole. **Explain** how to find a percentage of a whole by using the decimal equivalent to multiply the whole (e.g., 35% of 120 = 0.35 × 120) or by finding 10%, 5%, or 1% of the whole and using operations (e.g., finding 35% of 120 by finding 10%, multiplying this by 3 to get 30%, then adding half of 10% – 12 × 3 + 6 = 42).
	› add and subtract fractions with different denominators of up to a tenth, using equivalent fractions (e.g., $\frac{3}{4} + \frac{1}{3}$)	› add and subtract fractions with different denominators, using equivalent fractions	Demonstrate and **explain** renaming fractions, using ideas about equivalence and by finding HCFs and LCMs.
	› add and subtract decimals to three decimal places, with an emphasis on estimating before calculating	› add, subtract, and multiply decimals, with an emphasis on estimating before calculating	**Connect** methods for operating on whole numbers with operating on decimals, making an estimate before calculating. **Investigate** situations where decimals are compared and the differences between them found (e.g., sporting event times and distances). Have students practise decoding and solving word problems and **representing** them as equations.
	› use proportional reasoning to explore multiplicative relationships between quantities (e.g., "If there are 3 red for every 7 blue balls, how many balls are there altogether when there are 18 red balls?")	› use proportional reasoning to share with unequal proportions (e.g., "We have 100 stickers to share. For every 1 sticker I get, you get 3. How many do we each get?")	**Investigate** proportional reasoning in situations such as mixing paints, cooking from recipes, and sharing resources. **Represent** situations involving proportional reasoning using diagrams and comparison bar models. **Connect** proportional reasoning to multiplicative thinking and equivalent fractions.

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Financial mathematics	› calculate total cost and change for any amount of money	› create and compare weekly, monthly, and yearly finance plans (e.g., saving plans, phone plans, budgets, and 'buy now, pay later' services)	**Explain** and **justify** 'best deals', considering personal priorities. Represent financial plans for practical situations using digital tools such as spreadsheets. **Investigate** situations where there are financial percentage losses or gains (e.g., calculating discounts or profits, statistics in the media about growth or decline). **Connect** the ideas of loss and debt with integers. **Explain**, using worked examples, finding a percentage discount by subtracting from the whole or by multiplying the whole by a decimal fraction (e.g., a 35% discount on $180 = $180 – (0.35 × $180), or 0.65 × $180).
	› apply percentage discounts to whole-dollar amounts.	› apply percentage discounts.	

Algebra

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Equations and relationships	› form and solve one-step linear equations (e.g., t + 7 = 12, 2s = 14)	› form and solve one- or two-step linear equations (e.g., 5s + 3 = 18)	Have students practise writing equations to **represent** word problems. Demonstrate solving one- or two-step equations and using the inverse operation to check findings.
	› find the value of an expression or formula, given the values of variables (e.g., "Calculate w + 12 when w = 4")	› find the value of an expression or formula, given the values of variables	**Investigate** variable values in practical situations with familiar formulae (e.g., for area, volume, speed). Have students practise substituting measurements or given values into formulae.

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Equations and relationships	› describe and use the commutative, distributive, and associative properties of operations (e.g., a × b = b × a)	› simplify algebraic expressions involving sums, products, differences, and single brackets (e.g., using the distributive property, 2(x + 3) + 1 = 2x + 6 + 1 = 2x + 7)	**Represent** terms in algebraic expressions using algebra tiles. Represent algebraic expressions and equations using the conventions of algebra (e.g., 3 × b or b × 3 is written as 3b). At year 8, explain how to simplify algebraic expressions by collecting like terms together. At year 8, **investigate** systematic expansion approaches, including expansion tables, connecting to the distributive property.
	› identify the constant increase or decrease in a linear pattern, use variables and algebraic notation to represent the rule in an equation, and use the rule to make conjectures	› determine if a pattern is linear and, if it is, write the equation for the pattern and use the equation to make conjectures	**Represent** a pattern using a table, model, or diagram, and use it to **generalise** a rule for the pattern. Use the rule and an XY graph to **justify** a conjecture for another term in the pattern. **Investigate** the history, meaning, and structure of growing patterns (e.g., tukutuku, other well-known patterns such as the Fibonacci sequence).
Algorithmic thinking	› create, test, and revise algorithms involving a sequence of steps and decisions.	› create, test, revise, and use algorithms to identify, interpret, and explain patterns.	**Connect** an algorithm with an operation such as the vertical-column method for multiplication or with the procedure for adding fractions. **Represent** algorithms using flow charts, numbered step-by-step instructions, or digital tools. Explore algorithms **by investigating**: › the formula function of a spreadsheet and the effect of changing the value of a variable in a formula (e.g., hourly wages) › sorting and filtering multivariate data › sorting numbers according to a set of instructions (e.g., the sieve of Eratosthenes) to find prime numbers › situations that can be described and tested (e.g., divisibility, the use of transformations in a shape pattern, converting between units of measurement) › creating, testing, and revising a set of instructions using a digital tool.

Measurement

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Measuring	› estimate and then measure length, area, volume, capacity, mass (weight), temperature, data storage, time, and angle, using appropriate units	› estimate and then measure length, area, volume, capacity, mass (weight), temperature, data storage, time, and angle, using appropriate units	**Connect** to benchmarks to make estimations. Have students practise the accurate use of rulers, scales, timers, protractors, thermometers, and measuring jugs in practical situations. **Represent** all written measurements with their units. Select appropriate tools and units for a situation, and **explain** and **justify** choices.
	› select and use an appropriate base measure (e.g., metre, gram, litre) within the metric system, along with a prefix (e.g., kilo-, centi-) to show the size of units	› select and use an appropriate base measure within the metric system, along with a prefix to show the size of units	
	› convert between metric units of length, mass (weight), and capacity, using whole numbers and decimals to express parts of a unit (e.g., 724 g = 0.724 kg)	› convert between metric measurement units, including square units	**Connect** measurement conversions with multiplying and dividing by powers of 10 (e.g., 2.05 L = 2050 mL). **Investigate** measurement conversion situations in which all four operations are applied to whole-number and decimal measures.
	› find speed, given distance and time	› find distance, given speed and time; or time, given distance and speed	**Investigate** the relationship between speed, distance, and time in practical situations, such as timing how long it takes to walk or run a certain distance. Have students practise substituting values into the speed formula. **Connect** finding the value of variables in the speed formula with solving algebraic equations and multiplication and division operations.
	› read, interpret, and use timetables and charts that present information about duration › convert between units of time, and solve duration problems that involve fractions of time	› read, interpret, and use timetables, charts, and results that present information about duration › convert times to a common unit, such as seconds or minutes, and use decimal units of time (milliseconds)	**Explain** how to plan journeys using timetables and charts. Draw on a range of examples, including digital tools. **Explain** methods of calculating duration (e.g., subtracting time), using worked examples. **Investigate** the use of decimal units (milliseconds) in situations where a more precise measurement is needed (e.g., sporting events).

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Perimeter, area, and volume	› calculate the perimeter and area of composite shapes composed of triangles and rectangles.	› calculate the volume of triangular prisms and shapes composed of rectangular prisms.	**Investigate** perimeter, area, and volume, including finding missing lengths, in practical situations. **Connect** calculations with factors, multiples, and the commutative and associative properties. **Represent** working for calculations using a clear layout and by sketching composite shapes to show partitioning. **Generalise** the formulae for finding the area of triangles and volume of triangular prisms, and have students practise substituting measurement values into them. Connect the formulae with spatial representations.

Geometry

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Shapes	› classify and name shapes based on their attributes (e.g., triangles, pyramids)	› describe triangles, quadrilaterals, and other polygons in relation to their sides, diagonals, and angles	Use and create a range of 2D and 3D shapes, including shapes that draw on tactile materials, diagrams, and digital tools. **Investigate** ways of classifying shapes, including by creating algorithms and using Venn diagrams and tables.
	› identify and describe angles at a point, angles on a straight line, and vertically opposite angles	› reason about unknown angles in situations involving angles at a point, angles on a straight line, vertically opposite angles, and interior angles of triangles and quadrilaterals	**Investigate** using digital tools and protractors to explore angles. **Investigate** unknown angles to **generalise** the following rules: › the sum of the angles round a point is 360° › the sum of the angles on a straight line is 180° › vertically opposite angles are equal › the sum of the interior angles of a triangle is 180° and of a quadrilateral is 360°. **Represent** the value of an unknown angle using an equation and angle notation.

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Spatial reasoning	› visualise, construct, and draw plan views for front, back, left, right, and top views of 3D shapes	› visualise and draw nets for prisms with a fixed cross section	**Represent** plan views and nets, using sketches on grid paper, digital tools, and physical models (e.g., blocks, cardboard nets). **Connect** to measurement procedures when creating sketches and models.
	› transform 2D shapes, including composite shapes, by resizing by a whole number or unit fraction	› recognise the invariant properties of 2D and 3D shapes under different transformations	**Explain** and demonstrate resizing a shape using a centre of enlargement within the shape. **Investigate** transforming shapes to **generalise** which properties (angles, side lengths, area, orientation) do not change under transformation, and test the resulting generalisations using tracing paper, rulers, and protractors. **Investigate** the meaning of kōwhaiwhai patterns and other artefacts, and describe the use of transformations in them.
Pathways	› interpret and communicate the location of positions and pathways using coordinates, angle measures, and the 8 main and halfway compass points (e.g., NE, which is 45° E from N).	› use map scales, compass points, distance, and turn to interpret and communicate positions and pathways in coordinate systems and grid reference systems.	Use maps of familiar and unfamiliar locations to: › explain and investigate the use of 4-digit grid references › calculate distances using scales › find efficient routes between destinations. **Connect** pathways to: › measurement procedures when finding angles and distances › proportional reasoning when using map scales › algorithms to describe routes between two points. **Investigate** the navigating techniques of Māori and Pacific voyagers for locating position and finding the direction of travel.

Statistics

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Problem	› investigate, using multivariate datasets, summary, comparison, time-series, and relationship situations for paired categorical data by: – posing an investigative question about a local community matter – making conjectures or assertions about expected findings	› investigate, using multivariate datasets, summary, comparison, time-series, and relationship situations by: – posing an investigative question about a local community matter – making conjectures or assertions about expected findings	Show, with student input, how to pose **investigative** questions, clearly identifying the variable, the group of interest, and the intent. **Connect** investigative questions with conjectures about expected findings.
Plan	› plan how to collect or source data to answer the investigative question, including: – determining or identifying the variables needed – planning how to collect data for each variable (e.g., how to measure it) or finding out how provided data was collected – identifying the group of interest or who the data was collected from – building awareness of ethical practices in data collection by strategic questioning of data-collection questions or methods		**Explain** and discuss ethical practices for the collection and use of data. **Represent** planning using a planning tool to outline methods of data collection, 'who' and what to measure, and how. Show, with student input, how to pose data-collection and survey questions. **Explain** the variables and group or groups of interest in secondary datasets. **Investigate** how survey and data collection questions can be misinterpreted, leading to unreliable data.
Data	› collect primary data or gather information about variables in sourced data, create a simple informal data dictionary, and check for errors	› collect or source data, including: – checking for errors and following up and correcting them when possible – creating an informal data dictionary with information that will help others know about the context	Show, with student input: › a range of data-collection and recording methods › how to identify errors in data, **connecting** to the context and explaining why they are errors › how to update primary data when correctable errors are found. **Connect** multiple variables for individuals, explaining that most datasets use a table design in which each row focuses on an individual and each column includes the data on multiple individuals for one variable.

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Analysis	› create data visualisations for the investigation › make statements about the data, including its features and context, in descriptions of distributions	› create data visualisations for the investigation, using multiple visualisations to provide different views of the data › make statements about the data, including its features and context, in descriptions of distributions	Show, with student input, how to: › **represent** data using dot plots, bar graphs, frequency tables, time-series graphs, two-way tables or graphs, scatter plots, fractions, proportions, and percentages, creating them at first by hand and then with digital tools › read the data, read 'between' the data, and read 'behind' the data › describe what is seen in the data visualisations, recognising that data are numbers with context, and the context includes variables of interest, groups of interest, counts or proportions for categorical variables, and values and units for numerical variables › compare data visualisations of the same variable for different groups by looking at similarities and differences. **Explain** how different data visualisations have different features and how to describe them in context (e.g., in relation to the middle, distributional shape, joint and conditional proportions, long-term trends).
Conclusion	› communicate findings in context to answer the investigative question, using evidence from analysis and comparing findings to initial conjectures or assertions and their existing knowledge of the world	› communicate findings in context to answer the investigative question, using evidence from analysis, considering possible explanations for findings, and comparing findings to initial conjectures or assertions and their existing knowledge of the world	Show, with student input, how to: › choose the best descriptive statements that answer an investigative question › explore explanations or interpretations of findings that connect to the context of the situation under investigation › prepare and present succinct findings › explain and justify whether or not findings align with initial conjectures or assertions, and if what was found makes sense given what is known about the situation.
Statistical literacy	› evaluate the findings of others to check if their claims or statements are supported by the data visualisations they use.	› evaluate the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are reasonable.	Show, with student input, how to: › identify misleading data visualisations, match others' data visualisations with their statements, and check the claims made by others › **explain** and **justify** others' statements about the findings of statistical investigations and the process for collecting data, using interrogative questions.

Probability

	During year 7 *Informed by prior learning, teach students to:*	During year 8 *Informed by prior learning, teach students to:*	Teaching considerations
Probability investigations	› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by: – posing an investigative question – anticipating what outcomes are possible and which of them are more or less likely to occur – identifying and systematically listing possible answers to the investigative question – collecting and recording data – creating data visualisations for the distribution of observed outcomes – describing what these visualisations show – finding the probability estimates for the different outcomes – answering the investigative question – identifying similarities and differences between their findings and those of others – reflecting on anticipated outcomes – comparing findings from the probability experiment and associated theoretical probabilities, as appropriate	› plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by: – posing an investigative question – anticipating what outcomes are possible and which of them are more or less likely to occur – identifying and systematically listing possible answers to the investigative question – collecting and recording data – creating data visualisations for the distribution of observed outcomes and for all possible outcomes for theoretical probability models, where they exist – describing what these visualisations show – finding the probability estimates for the different outcomes – proposing possible theoretical outcomes and associated probabilities, for situations where no theoretical model exists – answering the investigative question – identifying similarities and differences between their findings and those of others – reflecting on anticipated outcomes – identifying similarities and differences between findings from the probability experiment and associated theoretical probabilities, as appropriate	**Investigate**, using the statistical enquiry cycle, games of chance, other everyday chance-based situations, patterns in possible outcomes, and theoretical and experimental distributions. **Represent** probability outcomes (theoretical and experimental) using lists, tables, tree diagrams, tally charts, visualisations of distributions, words, numbers, and technology. **Explain** how to describe and use probability concepts (e.g., outcomes, events, trials, models; theoretical and experimental probability; with and without replacement; the law of large numbers; probability estimates, probability distributions; chance, randomness, and variation). **Connect** anticipated outcomes with theoretical and experimental distributions. **Connect** probabilities with proportional reasoning, fractions, and percentages, and with relative frequencies from data investigations.
Critical thinking in probability	› identify, explain, and check others' statements about chance-based investigations, referring to evidence.		Show, with student input, how to: › match the results of others' chance-based investigations with statements › **explain** and **justify** the statements made by others about their findings from chance-based investigations, using interrogative questions.

The language of mathematics and statistics: Phase 3

	Year 7 *Students will know the following new words:*		Year 8 *Students will know the following new words:*	
Number	› discount › divisibility rule › exponent › highest common factor (HCF) › integer › lowest (least) common multiple (LCM)	› simplify › square root	› benchmark fraction › budget › composite number › cube number › financial plan › percentage increase or decrease	› powers of 10 › prime number
Algebra	› coefficient › coordinate › expression › like term › line graph › reciprocal › X axis, horizontal axis	› XY plane › Y axis, vertical axis	› expand › linear relationship › rate of change › substitute	
Measurement	› composite shape › digital › duration › formula › rate › speed		› millisecond › square unit	

	Year 7 *Students will know the following new words:*		Year 8 *Students will know the following new words:*	
Geometry	› complementary or supplementary angle › scale factor		› cross section › diagonal › exterior angle › grid reference › invariant property	
Statistics	› continuous data › critique › interpret › measure of centre (mean, median, mode)		› distribution › long-term trend › multivariate data set › time series	
Probability	› dependent, independent › event › experiment › experimental or theoretical probability › trial		› distribution › misconception › model › random	